Starred exercises are optional.

1) Consider the relation over words over the alphabet $\Sigma = \{a, \ldots, z\}$:

$$R := \{(w, v) \mid \forall x \in \Sigma : x \text{ occurs in } w \text{ iff } x \text{ occurs in } v\}$$

and let $<$ be the usual (strict) alphabetic order on letters $a < b < \ldots < z$.

- Show that R is an equivalence relation.
- Describe in words what are the equivalence classes of R, give an example of an equivalence class having exactly one element, and an example of an equivalence class having more than one element.
- Give a system of representatives for (the equivalence classes of) R. (Don't forget to argue that each representative is indeed unique in its equivalence class.)
- Does the lexicographic order $<_{lex}$ *preserve* the equivalence classes, that is, is it true that if $w <_{lex} v$ and $w \, R \, w'$ then also $w' <_{lex} v$?
  Hint: try to find words w, w', v such that $w <_{lex} v$ and $w \, R \, w'$, but $w' >_{lex} v$.

2) Assume the lecture of DS is attended by 168 students. Three fourth of them are experts in Haskell, half of them in Java and two thirds of them in C. Furthermore, 65 students master Haskell as well as Java, 89[1] Haskell as well as C and 39 are experts in Java as well as C.

   a) Assume every student knows Haskell, Java or C. How many students are experts in all three of theses languages?

   b) Assume only 29[1] students are experts in all three of these languages. How many students know neither Haskell nor Java nor C?

3) a) Suppose there are 20 injective functions from A to C, and 60 injective functions from B to C. How many injective functions are there from A to B, and how many elements does C have?

   b) Suppose the set M has 2 elements. How often do we need to repeat the powerset construction on M (i.e. what is the number n in $\overbrace{\mathcal{P}(\ldots(\mathcal{P}}^{n}(M))\ldots)$ needed) to obtain a set having cardinality greater than the number of inhabitants of Innsbruck, the number of people on earth, respectively the number of protons in the (observable) universe?

   c) Show that there are no undirected graphs having exactly 7 nodes all having an odd degree.

4*) a) Give a bijection f between $\mathbb{N}$ and $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ and compute $f(345)$ and $f^{-1}(3, 4, 5)$.

   b) Describe in words a procedure for enumerating the set of all (single class) Java programs (showing that that set is countably infinite).

[1] Updated four times on 19-11-2020, 18:00-00:00, to make numbers consistent.