Homework

Mark your solved exercise in OLAT and also upload your solution as a PDF.

1. The deterministic Nelson–Oppen method requires stably infinite and convex theories. What can go wrong in the deterministic Nelson–Oppen method in case of non-convex theories (without the case-splitting extension)?

   (a) The method outputs ”satisfiable” for an unsatisfiable formula. (1p.)

   (b) The method outputs ”unsatisfiable” for a satisfiable formula. (1p.)

   For each of the cases provide an answer with a justification, i.e., if something can go wrong provide a witness, e.g., a formula in the theories of LIA and EUF, or give a brief explanation why that case cannot happen.

2. Consider the following formula for applying Cooper's method.

   $$\varphi := \forall x\, y\, z. 14x - 21y + 11 \neq 63z$$

   (a) Convert φ into an equivalent formula ψ where the quantification of z is removed. Write down each step that is performed in Cooper's method. You can use intermediate arithmetic simplifications and should simplify the final formula. (1p.)

   (b) The formula that you have computed in the previous exercise might be:

   $$\psi := \neg\exists x\, y. 63 \mid 14x - 21y + 11 \equiv \varphi$$

   Transform ψ further into an equivalent quantifier free formula χ using Cooper's method. Use the optimizations from the lecture! (1p.)

   (c) Finally compute whether χ is valid, e.g., by writing a computer program that evaluates χ. (1p.)

3. In the lecture it was just briefly stated that without divisibility predicates there is no quantifier-free LIA formula φ such that φ is equivalent to $\exists y. 2y = x$.

   Think about a proof of such a statement and describe the idea of your proof informally. (2p.)

4. In order to simplify the proof of PSPACE-completeness of QBNF, the standard definition of acceptance of a deterministic Turing machines was changed. The original acceptance condition is defined as reachability of some configuration (t, y, n) where t is the accept state, y is an arbitrary tape content and n an arbitrary position on the tape.

   The alternative acceptance condition is defined as reachability of the configuration $(t, \vdash \llcorner\!\lrcorner^{\omega}, 0)$.

   Show that Turing machines using the original acceptance can be simulated by Turing machines using the alternative acceptance condition, i.e., define a translation of an arbitrary TM M into some machine M' such that for every input x, x is accepted by M using the original acceptance condition if and only if x is accepted by M' using the alternative acceptance condition. You may assume that the input alphabet does not contain the blank-symbol $\llcorner\!\lrcorner$.

   (a) Describe the idea of the translation informally. (1p.)

   (b) Describe the translation formally. (2p.)

   You do not have to prove that your translation is correct.