universität innsbruck

Constraint Solving	WS 2021	LVA 703305
Week 9		December 9, 2021

Homework

Mark your solved exercise in OLAT and also upload your solution as a PDF before December 9, 6am.

1. Apply Ferrante and Rackoff's method in order to convert the following formula into an equivalent quantifier free formula. (2p.)

   $$\varphi := \forall x. \exists y. 2y > 3x \wedge 4y < 8x + 10$$

   Remark: perform basic arithmetic simplifications after having eliminated y, and before eliminating x. However, perform the elimination of x also via Ferrante and Rackoff's method and do not use "obvious arithmetic reasoning" to immediately see the truth value of φ.

2. Prove soundness of the essential step of Ferrante and Rackoff's method, i.e., the equivalence between $\exists x.\varphi_3(x)$ and φ_4, cf. slide 9.

   Hint: Perform a similar argumentation as in the soundness proof of Cooper's method.

   (a) Show that whenever φ_4 is satisfiable then so is $\exists x.\varphi_3(x)$. (1p.)

   (b) Show that whenever $\exists x.\varphi_3(x)$ is satisfiable then so is φ_4. (2p.)

3. Consider the set I of linear inequalities over $\mathbb{Q}$:

   $$\begin{aligned} y - 2x &\leqslant 4 \\ x - 2 &\leqslant 3y \\ x - 2y &\geqslant 0 \\ x + 2y &\leqslant -2 \end{aligned}$$

   (a) Solve I using the simplex method, by hand on paper. Check that the solution you find is indeed a solution. (2p.)

   (b) Visualise I and the simplex solution process on it in a two-dimensional diagram (draw x, y within the range $[-3, \frac{1}{2}]$). (1p.)

4. In the context of Nelson–Oppen's method to combine theories, it is essential to determine the set of implied equations. Provide an algorithm based on the Simplex method that given a conjunction of linear inequalities φ and two variables x and y, the algorithm decides whether φ implies $x = y$. (2p.)

   Hint: You may assume that the Simplex method can be applied to arbitrary conjunctions of linear inequalities, i.e., both strict and non-strict inequalities.