Homework

Mark your solved exercise in OLAT and also upload your solution as a PDF before December 16, 6am.

1. Consider the set S of linear inequalities:

$$x - 2y < 1 \qquad x - y \geqslant 2 \qquad x < 4$$

   (a) Convert the inequalities into general form by introducing δ and slack variables – Do you really need a slack variable for $x < 4$? – and manually execute the simplex algorithm using Bland's rule on paper to find a solution; use the variable order $x < y < s_1 < \ldots$; provide intermediate results and indicate which variables have been used for pivoting. (2 pt.)

   (b) The solution of (a) will most likely contain elements in $\mathbb{Q}_\delta$. Design an algorithm for the following task: let S be an arbitrary set of inequalities and S' be the corresponding set of non-strict inequalities that has been obtained from S via the introduction of δ; convert a $\mathbb{Q}_\delta$-solution of S' into $\mathbb{Q}$-solution of S. Apply the algorithm on your solution of part (a). (2 pt.)

   (Remark: You may test your algorithm further on variations on S obtained by replacing the last inequality $x < 4$ by $x < q$ for values q getting closer and closer to 3.)

2. Argue that the unsatisfiable core that is generated by the simplex algorithm is minimal, cf. slide 19. (2 pt.)

3. Determine satisfiability of the following LRA formula φ using DPLL(T) and the incremental simplex implementation from the lecture.[1] You should use standard Boolean unit propagation as well as theory propagation, i.e., determine sets of implied literals from a partial Boolean assignment via the theory solver. (2 pt.)

   Hint: For this example you never have to perform any guesses, i.e., do not apply the inference rule for a Boolean decision.

[1] `http://cl-informatik.uibk.ac.at/teaching/ws21/cs/simplex.tgz`

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\begin{aligned}
\varphi &:= c_1 \wedge c_2 \wedge c_3 \wedge (c_4 \vee c_5) \wedge (c_6 \vee c_7) \\
c_1 &:= 0.5x - 20y + 2z \leqslant 5 \\
c_2 &:= 17y + 4z \leqslant 7 \\
c_3 &:= x + y + z \leqslant 100 \\
c_4 &:= -2x + y - z \geqslant 20 \\
c_5 &:= 3x + 14y + 7z \geqslant 500 \\
c_6 &:= x - 2y + z > 10 \\
c_7 &:= 2x - 3y + 2z \geqslant 100
\end{aligned}
$$

4. Consider a factory that produces three products A, B, and C. Each product requires a different combination of raw materials M, N, O, and P, i.e., for producing one unit of A one requires 3 N and 4 O, product B requires 2 M, 2 N, and 4 O, and product C requires 3 M, 1 N, 6 O, and 2 P. The quantities of the available materials of the company are 50 M, 200 N, 200 O, and 100 P.

   (a) Is it possible to produce products with a total value of 60000 EUR, if the values of A, B, and C are 1400 EUR, 1500 EUR, and 2200 EUR, respectively? Use the non-incremental simplex implementation to figure out the answer. (1 pt.)

   (b) Determine the optimal value as indicated on slide 23. Since the provided implementation does not permit to change bounds, just invoke the simplex algorithm several times from scratch. Find a suitable strategy to find the optimal solution quickly. (1 pt.)