Starred exercises are optional.

1) Consider a directed multigraph with a set of vertices $V = \{A, B, C, D\}$ and a set of edges $E = \{0, 1, 2, \ldots, 8\}$. The functions src and tgt are given by following table:

e	src	tgt
0	C	C
1	C	A
2	A	C
3	D	A
4	C	B
5	B	D
6	B	B
7	C	B
8	D	C

a) Visualize the graph.

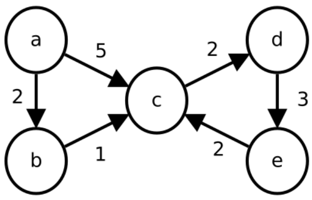
b) Name all immediate predecessors and immediate successors of node A.

c) What is the indegree and the outdegree of node B.

d) Is the multigraph strongly connected?

e) Give the adjecency matrix of the graph.

2) Consider the following weighted digraph.



a) Use the Floyd's algorithm to compute the shortest paths among the nodes. Enumerate the nodes alphabetically, that is, the first row in a matrix corresponds to node a, second to b, and so on. Compute all the intermediate matrices A_0, A_1, ..., A_5.

b) Do the same as in a) but reverse the node enumeration, that is, the first matrix row will correspond to node e, the second to d, and so on.

c) Are the matrices in a) and b) the same or do they differ? Try to explain what is the meaning of $(A_k)_{i,j}$ for $k = 1, \ldots, 5$ (the element at the position (i, j) in matrix A_k).

3) A *cycle* is a non-empty path (i.e. at least one edge) whose source and target are the same.

An *arborescence* is a directed graph G that contains a designated vertex v_0 (called its *root*) such that for every vertex v in G, there exists exactly one path from v_0 to v.

a) Show that an arborescence cannot contain a cycle.

b) Let G be an arborescence whose root v_0 has k immediate successors $v_1, \ldots, v_k$. Let G' be the graph obtained by removing v_0 and all edges originating from v_0 from G. Show that G' consists of k disjoint subgraphs $G_1, \ldots, G_k$ where each G_i is an arborescence with v_i as its root.

4∗) Consider again the graph from Exercise 2.

a) Change the weight between d and e to -1. Will Floyd's algorithm still work correctly?

b) Change the weight between d and e to -5. Will the algorithm work now?

c) Try to state under which circumstances negative weights can be allowed, and Floyd's algorithm still produces correct results.