1) Imagine your PS-teacher has baked some cookies for the proseminar. If the cookies were given to three of you (shared equally) there would be one cookie left. If the cookies were given to four of you (shared eqally) there would be two cookies left. If the cookies were partitioned to all participants (25 students) of the proseminar then four would be left. Further the PS-teacher give you the hint that there was definitely no time to bake more than 300 cookies. How many cookies were baked?

2) a) Let $C = \{(x, y) \mid x, y \in \mathbb{R},\ x^2 + y^2 = 1\}$ be the unit circle in 2 dimensions. What is $|C|$?

   b) Show that if $f : A \to B$ is surjective then $|A| \geq |B|$.

   c) Show that $|2^{\mathbb{N}}| = |4^{\mathbb{N}}|$. Recall that $2 = \{0, 1\}$ and analogously $4 = \{0, 1, 2, 3\}$ (or more generally any set with 2 elements, or with 4, respectively).

   d) Generalise the previous subexercise: under what conditions does $|A^{\mathbb{N}}| = |2^{\mathbb{N}}|$ hold? Give a sketch of the proof!

3) All the following exercises should be computable by hand using appropriate results from the lecture. Don't use anything stronger than a 16-bit calculator.

   a) Compute $(411^{4110} + 410) \bmod 4111$ given that 4111 is a prime.

   b) Compute $\gcd(77, 111)$ using the two variants of the Euclidean algorithm from the lecture (the difference and the remainder variants). How many steps do you need with each of the variants?

   c) Is the congruence class $\overline{77}$ modulo 111 invertible? If so, compute its inverse. Then find some n such that $\overline{77}$ modulo n is not invertible.

   d) Compute $111^{60} \bmod 77$ using the Euler's theorem.

4*) In the following exercise, provide a *constructive* definitions of the functions which allow you to compute function values from arguments (as if you were to write a program).

   a) Construct a bijection f_3 from $\mathbb{N}^3$ to $\mathbb{N}$. (Hint: Extend the *dove tailing* function from the lecture.)

   b) Generalize the above function f_3 to a bijection f_n from $\mathbb{N}^n$ to $\mathbb{N}$ $(n > 3)$.

   c) Let $\mathbb{N}^\star$ be the set of all finite sequences of natural numbers which don't end with 0. Provide a constructive definition of a bijection from $\mathbb{N}$ to $\mathbb{N}^\star$. List the first 10 sequences in your enumeration.

   d) Let A be the set of all finite words over the alphabet $\{\texttt{a},\texttt{b}\}$ (including the empty word). Construct a bijection from A to $\mathbb{N}$.