1) Give a Θ estimate for each of the following recurrences (1), (2) and (3), for $n > 1$:

$$T_1(n) = T_1(\tfrac{n}{3}) + c \tag{1}$$
$$T_2(n) = 7 \cdot T_2(\tfrac{n}{6}) + 3n^2 \tag{2}$$
$$T_3(n) = 32 \cdot T_3(\tfrac{n}{2}) + n^4 \tag{3}$$

2) Consider binary trees which store natural numbers in their leaves, and a function which sums the leaf values, for example, in a Haskell-like notation:

```
sum (Leaf a) = a
sum (Node l r) = (sum l) + (sum r)
```

Assume that the time complexity of function `+` is $\Theta(1)$.

a) Explain why the Master theorem can not be used to compute the time complexity of `sum` for an *arbitrary* tree.

b) Describe a class of trees which allows you to use the Master theorem to compute the time complexity of `sum`. Compute the complexity `sum` on this class of trees.

3) Consider the following recurrence defined for $n = 2^k$ whenever $k > 0$.

$$T(n) = \begin{cases} \log_2(n) & \text{if } n \leq 4 \\ T(\frac{n}{2}) + 2 \cdot T(\frac{n}{4}) & \text{otherwise (if } n = 2^k) \end{cases}$$

Compute the values $T(n)$ for the first (smallest) 10 admissible values of n. Based on this observation, guess the closed form of T (a non-recurrent equivalent of T) and prove your guess to be correct.

4) Consider the following recurrence $T : \mathbb{N} \to \mathbb{N}$:

$$T(0) = 0 \qquad T(n) = T(\lfloor \tfrac{n}{3} \rfloor) + T(\lfloor \tfrac{n}{4} \rfloor) + n \quad \text{for } n \geq 1$$

What can you say about the asymptotic growth of T?