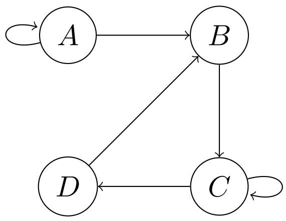
1) Consider the relation R on $\{A, B, C, D\}$ whose graph is:



Use Warshall's algorithm to compute the transitive closure R^+ of R.
Explain the connection between the transitive closure matrix of R and the distance matrix of R (if the edges were labeled with distances).

2) The equivalence of fractions is defined by following relation on $\mathbb{Z} \times (\mathbb{Z}\backslash\{0\})$:

$$(a, b) \sim (c, d) :\Leftrightarrow ad = bc$$

Show that this relation is reflexive, irreflexive, symmetric, anti-symmetric and transitive or give a counterexample.

Note: In the following, R^s denotes the symmetric closure of R.

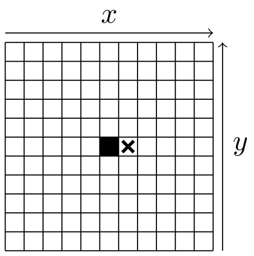
3) We define the following relations $R, S \subseteq \mathbb{Z}^2$ on points with integer coordinates:

$$R := \{((x, y), (x + 1, y) \mid x, y \in \mathbb{Z}\} \qquad S := \{((x, y), (x, y + 1)) \mid x, y \in \mathbb{Z}\}$$

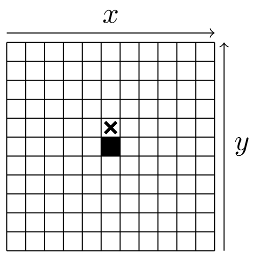
Informally, $p_1\ R\ p_2$ holds iff p_2 is one step to the right of p_1 and $p_1\ S\ p_2$ holds iff p_2 is one step above p_1.

We now visualise these relations as follows: we draw a grid where each cell represents a point in $\mathbb{Z}^2$. The cell in the centre (marked in black) represents a point p_1. We now mark all points p_2 such that $p_1\ R\ p_2$ with a cross.

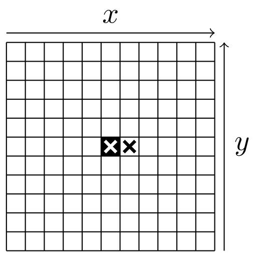
R:



S:



$R^=$:



Draw similar pictures for the following relations:

a) R^+ b) R^s c) $(R^=)^2$ d) $(R^* \cup S^*)^s$
e) $(R \cup S)^*$ f) $(RS)^*$ g) $(R^=S)^+$ h) $(R^*S^*)^*$

4*) Which of the following statements are true for **all** relations R and S? Give a counterexample for the false ones! For the true ones, give an informal explanation for why they hold.

a) $(R^s)^* = (R^*)^s$

b) $(R^=)^n = (R^n)^=$

c) $(R \cup S)^* = (R^* S^*)^*$

d) $(R \cup S)^* = R^* \cup S^*$

e) $(R \cap S)^* = R^* \cap S^*$

f) $(RS)^* = (SR)^*$

Hint: When looking for counterexamples, consider the relations R and S from ex. 3) and the relation $T := \{(x, -x) \mid x \in \mathbb{Z}\}$ or try to come up with a suitable small graph.