

Please note: Be very clear about what kind of induction you are performing and on what!

- 1) Prove by induction: For any natural numbers x and y , there exist integers a and b such that $ax + by = \gcd(x, y)$.

You may use without proof the following facts for any $x, y \in \mathbb{N}$:

- $\gcd(x, y) = \gcd(y, x)$
- $\gcd(x, 0) = x$
- $\gcd(x, y) = \gcd(x, y - x)$

- 2) Find and prove a relationship between the number of connected components in an undirected forest and its number of nodes and edges?

Hint: Look at the lemma proved in the lecture for trees and try some example forests.

- 3) Show that a undirected finite graph G is a forest if and only if every non-empty subgraph of G contains a vertex of degree ≤ 1 .
- 4*) Prove by induction: Let x, y be words over some alphabet Σ such that $xy = yx$. Then there exist a word z and natural numbers m, n such that $x = z^m$ and $y = z^n$.