1) We compute the transitive closure:

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

Imagine the edges were labled with distances and we computed Floyd's algorithm to compute the shortest paths among the nodes as it was done in exercise 2 of the 1st sheet. The matrices in the computation above can be obtained by mapping ∞ to 0 and natural numbers to 1. Note: Since R^+ is not necessarily reflexive (like the reflexive transitive closure R^*), we normally have to treat the diagonal separately.

2)

$$M = \mathbb{Z} \times (\mathbb{Z} \backslash \{0\})$$
$$R := \{((a,b),(c,d)) \in M^2 \mid ad = bc\}$$

reflexive: $(a,b) \in M \Rightarrow ((a,b),(a,b)) \in R$ since $ab = ab$
not irreflexive: It is reflexive.
symmetric: $(a,b),(c,d) \in M$ and $((a,b),(c,d)) \in R \Rightarrow ((c,d),(a,b)) \in R$ since $ad = bc \Leftrightarrow cb = da$
not anti-symmetric: $((1,2),(2,4)) \in R$ and $((2,4),(1,2)) \in R$ because $4 = 4$ but $(1,2) \neq (2,4)$
transitive: $(a,b),(c,d),(e,f) \in M$, $((a,b),(c,d)) \in R$ and $((c,d),(e,f)) \in R \Rightarrow ((a,b),(e,f)) \in R$ since $ad = bc$ and $cf = de$ we have $adcf = bcde$. When we cancel equal terms on both sides of the equation we get $af = be$.

3)