

Homework

1. Consider the set I of linear inequalities over \mathbb{Q} : (2 P)

$$y - 2x \leq 4 \qquad x - 2 \leq 3y \qquad x - 2y \geq 0 \qquad x + 2y \leq -2$$

- (a) Solve I using the simplex method, by hand on paper. Check that the solution you find is indeed a solution.
- (b) Visualise I and the simplex solution process on it in a two-dimensional diagram (draw x, y within the range $[-3, \frac{1}{2}]$).
2. Consider the following problem: (2 P)

A chemical plant can produce n different products a_1, \dots, a_n . It can sell a product a_i for a price per liter of $\text{price}(a_i)$ euros. All the products are made from m different raw ingredients e_1, \dots, e_m . The volume of ingredient e_i used to produce one liter of a product a_j is defined as $\text{ing}(e_i, a_j)$. The company currently has $\text{stock}(e_i)$ liters of ingredient e_i in stock. Is it possible for the company to produce at least R euros worth of products given its current stock of ingredients?

- (a) Assume that the $\text{price}(a_i)$, $\text{ing}(e_i, a_j)$ and $\text{stock}(e_i)$ is known for all products and ingredients. Find a LRA formula φ which encodes the problem. Note that also parts of a product can be sold, e.g., half a liter.
- (b) Test your encoding using an SMT solver for the instance of the problem with three products a_1, a_2, a_3 and three ingredients e_1, e_2, e_3 . The current stock of ingredients is $\text{stock}(e_1) = 50$, $\text{stock}(e_2) = 200$, $\text{stock}(e_3) = 100$, and the prices of products as well as ingredient required to produce a product are

i	$\text{price}(a_i)/\text{euro}$	$\text{ing}(e_1, a_i)$	$\text{ing}(e_2, a_i)$	$\text{ing}(e_3, a_i)$
1	120	0	6	2
2	140	2	2	4
3	250	4	1	5

Can the plant produce products worth more than 5600 euros? How about 5650 euros?

3. Extend the algorithm for difference logic to arbitrary constraints, i.e., including strict inequalities, such as the following ones.

$$x - y < -3 \qquad x - w \leq -4 \qquad y - z < 5 \qquad z - w \leq -2 \qquad z - x \leq -1$$

- (a) Show how strict inequalities can be added for \mathbb{Z} and test it on the example constraints. (1 P)
- (b) Show how strict inequalities can be added for \mathbb{Q} and test it on the example constraints. (2 P)
4. Assume we have a conjunction of inequalities C with variables ranging over \mathbb{Q} . These will be converted into the conjunction of non-strict inequalities C_δ over \mathbb{Q}_δ as indicated on slide 26.
- (a) Define an algorithm that given a satisfying assignment of C_δ using δ -rationals, produces an assignment over \mathbb{Q} which satisfies the inequalities in C . (2 P)
- (b) Test your algorithm on the following conjunction of inequalities (1 P)

$$3 \leq 2x + y \qquad -1 < -x + y \qquad -2 < -x - y$$

where the simplex algorithm finds the satisfying \mathbb{Q}_δ assignment $x = (1, 1)$ and $y = (1, -2)$.