$\boxed{1}$ Consider the formula

$$(1 \vee 2) \wedge (1 \vee \overline{2} \vee \overline{3}) \wedge (3 \vee \overline{4} \vee 5) \wedge (\overline{2} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee \overline{7} \vee 8) \wedge (6 \vee \overline{8} \vee 9) \wedge (3 \vee 6 \vee 10) \wedge$$
$$(\overline{8} \vee 11) \wedge (\overline{11} \vee 12) \wedge (\overline{9} \vee \overline{10} \vee \overline{11} \vee \overline{12})$$

and suppose a DPLL inference sequence reached the state $\overline{1}^d \, 2 \, \overline{3} \, 4^d \, 5 \, \overline{6} \, 10 \, 7^d \, 8 \, 9 \, 11 \, 12$.

[3]   (a) Construct an implication graph and give three different cuts together with the respective implied clauses. Which nodes are UIPs?

[3]   (b) Give an implied clause derived from a cut that has as few literals as possible. Derive this clause by resolution from the conflict.

[4] $\boxed{2}$ Consider the following EUF formula:

$$\mathsf{a} = \mathsf{b} \wedge \mathsf{c} = \mathsf{g}(\mathsf{a}) \wedge \mathsf{f}(\mathsf{a}, \mathsf{a}) = \mathsf{c} \wedge \mathsf{f}(\mathsf{b}, \mathsf{b}) = \mathsf{f}(\mathsf{c}, \mathsf{b}) \wedge \mathsf{f}(\mathsf{g}(\mathsf{a}), \mathsf{b}) \neq \mathsf{g}(\mathsf{a})$$

Use congruence closure to determine whether it is satisfiable.

$\boxed{3}$ Consider the following equality logic (EQ) formula φ:

$$\mathsf{a} = \mathsf{b} \wedge \mathsf{c} = \mathsf{d} \wedge (\mathsf{b} \neq \mathsf{c} \vee \mathsf{b} = \mathsf{e}) \wedge (\mathsf{a} \neq \mathsf{d} \vee \mathsf{b} \neq \mathsf{e}) \wedge (\mathsf{b} = \mathsf{c} \vee \mathsf{c} = \mathsf{a})$$

[1]   (a) Construct a propositional skeleton of φ.

[5]   (b) Use DPLL(T) to show that φ is unsatisfiable. Explain which DPLL(T) inference rules are used, which EQ problems appear and how they are solved using equality graphs.

$\boxed{4}$ Consider the formula φ:

$$(\neg x \vee \neg y) \wedge \neg x \wedge (x \vee y) \wedge (x \vee \neg y \vee z) \wedge (x \vee z) \wedge (x \vee \neg y) \wedge \neg z$$

[2]   (a) What are minUNSAT(φ) and maxSAT(φ)? Explain your answer.

[2]   (b) Determine a smallest unsatisfiable core (SUC) of φ.