Exercises 1 October 21, 2022

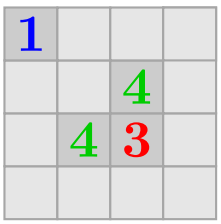
[2] 1 Consider the formula

$$(1 \vee \neg 3) \wedge (\neg 1 \vee \neg 4) \wedge (\neg 1 \vee \neg 3 \vee 4) \wedge (2 \vee 3 \vee 4) \wedge (\neg 2 \vee 4) \wedge (3 \vee \neg 4) \wedge (\neg 1 \vee 2 \vee 5)$$

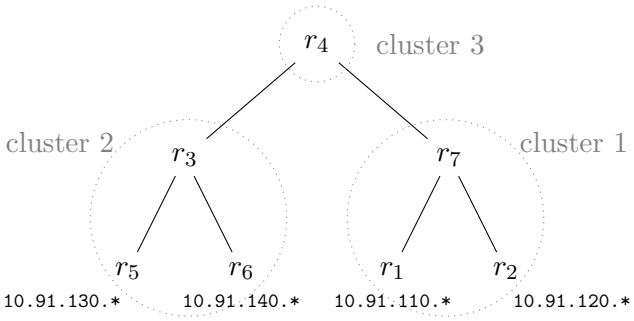
Give a DPLL inference sequence to determine its satisfiability.

[2] 2 Transform the formula $\phi = \neg(p \vee \neg(q \vee r)) \wedge (q \vee r)$ to CNF using (a) Tseitin's transformation and (b) the transformation by Plaisted and Greenbaum. Is the formula satisfiable?

[3] 3 Encode the following Minesweeper board as a SAT problem and solve it using Minisat or `z3py`. Is there more than one solution?



4 Consider the following network verification problem for seven routers arranged in this topology:



Thus, router r_3 is said to be *above* r_5 and r_6, r_7 is above r_1 and r_2, and r_4 is above all others. Moreover, every router belongs to a cluster, as shown. Some routers serves a virtual LAN (VLAN) directly, as indicated in the picture, e.g. r_5 serves the VLAN with IP addresses `10.91.130.*`.

Every router maintains a routing table. If a router gets a package with destination d, a router can deliver it directly if d is in its own VLAN. Otherwise, the router consults its routing table to determine where the package should be sent.

The current tables for r_1, r_3, and r_5 are as follows (where `or` means that the router can nondeterministically pick one destination):

```
Router r1:
if dst = 10.91.120.* then (r2 or r7) else
if dst = 10.91.130.* then r4 else
r7

Router r3:
if dst = 10.91.130.* then r5 else
if dst = 10.91.140.* then (r5 or r6) else
if dst = 10.91.110.* then r1 else
r4

Router r5:
if dst = 10.91.140.* then (r3 or r6) else
if dst = 10.91.110.* then r7 else
r4
```

For different reasons the routing should definitely satisfy the following requirements:

(C1) No router should send a package to itself.

(C2) Traffic directed to an address in a different cluster is forwarded to a router above.

(C3) Two routers in the same cluster and on the same level (i.e. not one above the other) route to the same destination.

[3] (a) Does the current configuration satisfy these requirements? Find an encoding to express this problem in SAT.

[3] ⋆(b) Implement your encoding and solve it using `minisat` or `z3py`.

Hint: The most important decision is how many variables to use, and what is their meaning. It must be at least possible to express that router r_i send a package with destination d to router r_j, for all possibilities of i, j, and d.

Exercises marked with a ⋆ are optional. Solving them gives bonus points.