	<u>Ç</u>	universität Institut for Informatik
SAT and SMT Solving	WS 2022	LVA 703147
Exercises 2		October 28, 2022

Consider the formula

1

[2]

[2]

 $\begin{array}{l} (\neg 1 \lor 2) \land (\neg 1 \lor \neg 2 \lor 3) \land (\neg 1 \lor \neg 2 \lor \neg 3 \lor 4) \land (\neg 2 \lor \neg 5) \land (\neg 3 \lor 5 \lor \neg 6) \land \\ (\neg 7 \lor 8) \land (\neg 8 \lor 9) \land (\neg 8 \lor \neg 9 \lor \neg 10) \land (\neg 12 \lor 13) \land (10 \lor \neg 12 \lor \neg 13 \lor \neg 14) \land \\ (\neg 8 \lor \neg 12 \lor \neg 13 \lor 14) \end{array}$

and suppose a DPLL inference sequence reached the state $1^d 2 3 4 \overline{5} \overline{6} 7^d 8 9 \overline{10} 11^d 12^d 13 \overline{14}$.

- [2] (a) Construct a conflict graph. Show all cuts which correspond to possible backjump clauses.
 - (b) Use the approach given on the slides of Week 2 to determine possible backjump clauses by resolution.

2 Consider the formula φ :

 $(1 \vee \overline{2}) \land (\overline{1} \vee 7) \land (3 \vee 4) \land (\overline{3} \vee \overline{4} \vee 8) \land (4 \vee 5 \vee 6) \land (\overline{8} \vee \overline{3})$

and the DPLL sequence consisting of textsfdecide and unit propagate steps leading to the following state:

$$\|\varphi \implies^* 1^d 73^d 4^d 85^d 6^d \|\varphi \tag{(\star)}$$

- (a) Use a **backjump** step to resolve the conflict. How many **backtrack** steps would be necessary to simulate it?
- [1] (b) Illustrate the two-watched literal scheme on the example: assign initial pointers to all clauses in φ and explain how they change in every decide and unit propagate step of (*).
- [3] 3 George the Gorgeous Gardener wants to plant onions, peas, broccoli, peppers, potatoes, tomatoes, and garlic in his field. The vegetables should be arranged in rows, e.g. like that:



He recently read in a book about organic gardening that one should not put "bad companion" plants next to each other: they may take away nutrients, or attract insects that can harm the neighbor. According to that

- (a) neither peppers nor broccoli should be planted next to onions,
- (b) peas should not be next to tomatoes or potatoes,
- (c) potatoes should not be next to peppers, onions, or tomatoes, and
- (d) garlic should not be next to peppers.

So the layout drawn above is not good because broccoli is next to onions. Encode the problem into a SAT formula to determine whether there is a layout which satisfies all conditions.

Hint: One possibility is to use 49 variables $x_{r,v}$ where $r \in \{1, \ldots, 7\}$ and $v \in \{\text{onions}, \ldots, \text{garlic}\}$, indicating that vegetable v is planted in row r.

- [3] \star 5 Anton, Bea, Carol, and Denis meet at the lobby of a hotel. There are some communication problems when they are trying to make conversation.
 - Among English, Russian, French, and German, each of them speaks exactly two languages.
 - They cannot find a language that everyone speaks, and there is only one language that three of them speak.
 - Nobody understands both French and German.
 - Anton does not speak English, but Bea and Carol need him as an interpreter.
 - Carol speaks German, Denis does not, but they can communicate directly.
 - Anton, Bea, and Denis cannot find a language they all can speak.

Who speaks which languages? Use a SAT encoding to find it out. You can use the Z3 bindings for Python (or another language), or write a DIMACS file and check with Minisat.

Exercises marked with a \star are optional, solving them gives bonus points.