SAT and SMT Solving | WS 2022 | LVA 703147

Exercises 4 | November 11, 2022

[2] $\boxed{1}$ Find two different minimal unsatisfiable cores and the SUC of the following formula:

$$(x \vee y \vee \neg z) \wedge \neg x \wedge (x \vee y) \wedge (x \vee \neg y \vee \neg z) \wedge (x \vee \neg z) \wedge (\neg x \vee \neg y \vee \neg z) \wedge (y \vee z) \wedge z$$

[3] $\boxed{2}$ Use the minUnsatCore algorithm to get a minimal unsatisfiable core of the following formula:

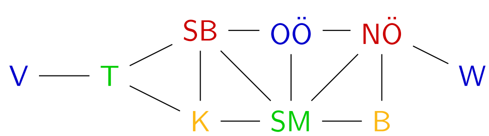
$$(x \vee y \vee z) \wedge \neg x \wedge (x \vee y) \wedge (x \vee \neg y) \wedge (y \vee \neg z) \wedge (\neg x \vee \neg w) \wedge (z \vee w)$$

$\boxed{3}$ The problem of k-vertex coloring for a given undirected graph assumes that there are k colors given and asks to find a color for every node such that no adjacent nodes have the same color.

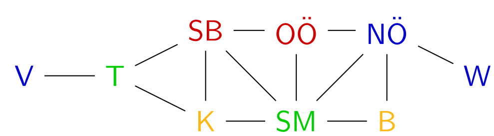
More precisely, given a graph $G = (V, E)$ a k-coloring for G exists if there is a function $c : V \to \{1, \ldots, k\}$ such that for all $(u, v) \in E$ we have $c(u) \neq c(v)$. The smallest k which admits a k-coloring of a graph G is called the *chromatic number* of G.

One application is coloring maps. For example, the following graphs are a valid (a) and an invalid (b) 4-coloring of the state map of Austria:

(a)



(b)



Being in NP, k-vertex coloring can be reduced to SAT, e.g. by using for every node n propositional variables $n_1, \ldots, n_k$ sucht that n_i becomes true if and only if node n has color i.

[2] (a) Use a SAT encoding to find a 3-coloring of the state map of Austria.

[3] (b) Use a SAT encoding to determine whether there is a 2-coloring of the state map of Austria. If there is no such coloring, get an unsatisfiable core to determine a minimal subgraph for which no 2-coloring exists.

[2] ⋆(c) Write a function which takes an undirected graph G and a number k and uses a SAT encoding to determine whether a k-coloring of G exists. (The graph can e.g. be given as an adjacency list, i.e., a list of edges.)

You can also do the above tasks for a different country (for fairness, let's exclude Liechtenstein, Monaco, and other dwarves with less than 9 states). If there is no 3-coloring, look for a 4-coloring: by the Four Color Theorem such a coloring exists for every map!