[3] **1** For the following sets of equations E, construct equality graphs and find all contradictory cycles to decide satisfiability:

a)

$$\begin{array}{llllll} v_1 = v_2 & v_1 = v_7 & v_2 = v_4 & v_2 \neq v_6 & v_2 \neq v_7 & v_3 \neq v_5 \\ v_4 = v_5 & v_5 = v_6 & v_5 = v_8 & v_6 \neq v_8 & v_7 = v_8 & v_7 \neq v_3 \end{array}$$

b)

$$\begin{array}{llllll} v_1 \neq v_2 & v_2 = v_3 & v_2 \neq v_4 & v_3 \neq v_6 & v_3 \neq v_8 & v_4 = v_5 \\ v_4 = v_6 & v_5 = v_6 & v_5 \neq v_7 & v_6 = v_8 & v_7 = v_1 & v_8 \neq v_2 \end{array}$$

2 Consider a signature with constants a, b, c, unary f and h, and binary g.

[1] a) Use congruence closure to determine whether for the equations E:

$$\begin{array}{lll} \mathsf{a} = \mathsf{b} & \mathsf{f}(\mathsf{a}) = \mathsf{b} & \mathsf{f}(\mathsf{b}) = \mathsf{c} \\ \mathsf{h}(\mathsf{a}) = \mathsf{h}(\mathsf{h}(\mathsf{b})) & \mathsf{f}(\mathsf{a}) = \mathsf{h}(\mathsf{b}) & \mathsf{f}(\mathsf{c}) = \mathsf{f}(\mathsf{h}(\mathsf{c})) \end{array}$$

it holds that $E \vDash_{\mathsf{EUF}} \mathsf{a} = \mathsf{c}$.

[1] b) Use congruence closure to determine whether the following set of literals is satisfiable:

$$\mathsf{f}(\mathsf{a}) = \mathsf{a} \qquad \mathsf{b} = \mathsf{g}(\mathsf{a}, \mathsf{b}) \qquad \mathsf{f}(\mathsf{f}(\mathsf{a})) = \mathsf{f}(\mathsf{g}(\mathsf{a}, \mathsf{b})) \qquad \mathsf{f}(\mathsf{a}) \neq \mathsf{c} \qquad \mathsf{c} = \mathsf{f}(\mathsf{b})$$

[1] c) Use congruence closure to determine whether the following set of literals is satisfiable:

$$\begin{array}{lll} \mathsf{f}(\mathsf{a}) = \mathsf{a} & \mathsf{f}(\mathsf{b}) = \mathsf{g}(\mathsf{f}(\mathsf{a}), \mathsf{b}) & \mathsf{f}(\mathsf{f}(\mathsf{a})) = \mathsf{g}(\mathsf{b}, \mathsf{b}) \\ \mathsf{g}(\mathsf{a}, \mathsf{b}) = \mathsf{b} & \mathsf{f}(\mathsf{g}(\mathsf{a}, \mathsf{b})) \neq \mathsf{g}(\mathsf{b}, \mathsf{b}) & \mathsf{f}(\mathsf{c}) \neq \mathsf{g}(\mathsf{a}, \mathsf{b}) \end{array}$$

[1] d) Consider the EUF formula from last week's exercise:

$$\mathsf{f}(\mathsf{b}) = \mathsf{g}(\mathsf{f}(\mathsf{a}), \mathsf{b}) \wedge \mathsf{f}(\mathsf{f}(\mathsf{a})) = \mathsf{g}(\mathsf{b}, \mathsf{b}) \wedge (\mathsf{f}(\mathsf{a}) = \mathsf{a} \vee \mathsf{f}(\mathsf{a}) = \mathsf{b}) \wedge \mathsf{g}(\mathsf{a}, \mathsf{b}) = \mathsf{b} \wedge (\mathsf{f}(\mathsf{b}) \neq \mathsf{b} \vee \mathsf{f}(\mathsf{g}(\mathsf{a}, \mathsf{b})) \neq \mathsf{g}(\mathsf{b}, \mathsf{b}))$$

Apply again DPLL(T) (or use your DPLL(T) sequence from last week), but now use congruence closure to check EUF-consistency of models.

[3] **3** In a 3×3 magic square the numbers $\{1, 2, \ldots 9\}$ are arranged in such a manner that all rows and all columns have the same sum. For a 4×4 magic square numbers between 0 and 100 are permitted (not all fit, of course), again all rows and columns should have the same sum. Use an SMT encoding to find out which of the following can be completed to magical squares.

	1	
4		

7		
		8

			87
	17		
		89	
19			