[3]   1   Use the Simplex algorithm to find a solution to the following system of linear inequalities in general form:

$$-x + y = s_1 \qquad s_1 \leq 1$$
$$-2x - y = s_2 \qquad s_2 \leq -4$$

2   Consider the following system of linear inequalities:

$$y \leq 4$$
$$-3x + y \leq -1$$
$$-x - y \leq -5$$
$$2x - y \leq 3$$

[1]   (a) Draw the solution space.
[2]   (b) Use the Simplex algorithm to find a solution.

[4]   3   The following is an instance of the travelling Santa Claus problem: The file `distances.py`, lists distances between 13 US cities in miles. Is there a tour (a circular route) to visit all cities below 9000 miles?

The following steps might be helpful:

(a) Create 13 integer variables $c_1, \ldots, c_{13}$ with the semantics that the route is $c_1 \to c_2 \to \ldots c_{13} \to c_1$, and $c_i = 1$ iff c_i is the first city in the list (New York), $c_i = 2$ iff c_i is the second city in the list (Los Angeles), etc.

(b) Formulate a constraints that all cities are visited, and no city is visited twice.

(c) Write a function `distance(c_i, c_j)` which takes two cities and returns an expression for the distance between city c_i and c_j. You can construct this expression as a big if-then-else expression, covering all 13×13 possibilities, looking up distances in the matrix from `distances.py`.

(d) Compute an expression for the total distance, and add a constraint demanding that it is below the given bound.

[4] ⋆ 4   Implement a Simplex solver for a set of linear inequalities of the form $A\vec{x} \leqslant \vec{b}$.

(This might admittedly be tricky. Details of the DPLL(T) Simplex algorithm can be found in the following technical report: Bruno Dutertre and Leonardo de Moura: *Integrating Simplex with DPLL(T)*, Technical Report SRI–CSL–06–01, SRI International, 2006.)