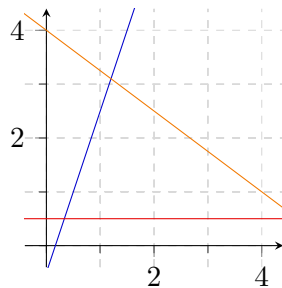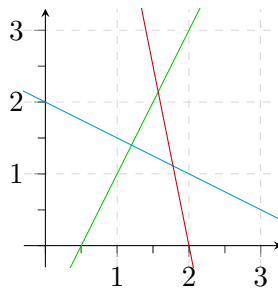$\boxed{1}$ Consider the following six sets of linear arithmetic constraints:
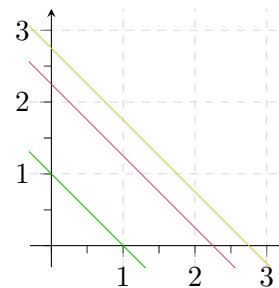
(1)
$$4 - x \leq 2y$$
$$y \leq 2x - 1$$
$$y \leq 10 - 5x$$

(2)
$$4y \leq 11 - 4x$$
$$9 - 4x \leq 4y$$
$$1 - x \leq y$$

(3)
$$2y \leq 4 - x$$
$$y \leq 2x - 1$$
$$y \leq 10 - 5x$$

(4)
$$4y \leq 11 - 4x$$
$$9 - 4x \leq 4y$$
$$y \leq 1 - x$$

(5)
$$4y \leq 16 - 3x$$
$$1 \leq 2y$$
$$2y \leq 6x - 1$$

(6)
$$1 \leq x$$
$$y \leq 2$$
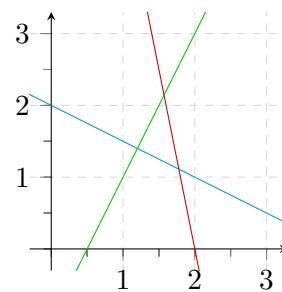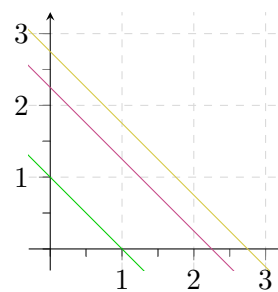$$x \leq y$$

(A)

(B)

(C)

(D)

(E)
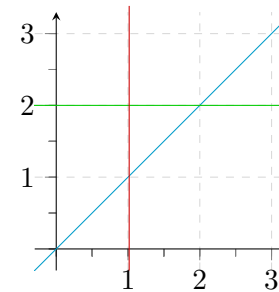
(F)

[1] (a) For each constraint set **(1)**–**(6)** pick one of the diagrams **(A)**–**(F)** to visualize its solution space.

[1] (b) Determine from the drawings which problems are satisfiable over $\mathbb{Q}^2$ and over $\mathbb{Z}^2$, and which problems are bounded.

[2] $\boxed{2}$ Apply Fourier-Motzkin elimination to the constraints **(2)**, **(5)** and **(6)** of the previous exercise, to check satisfiability over $\mathbb{Q}^2$.

3		Consider the following system of linear inequalities (cf. slide 13 of week 8):

$$3x - 2y \geq -1$$
$$y \leq 4$$
$$2x + y \geq 5$$
$$3x - y \leq 7$$

[2]   (a) Use (an implementation of) the DPLL(T) Simplex algorithm (e.g. this one) to find a solution to the problem in $\mathbb{Q}^2$. Derive a Gomory cut from the solution, and draw it in the diagram.

[1]   (b) Run DPLL(T) Simplex on this modified problem, and repeat the procedure until you found a solution in $\mathbb{Z}^2$.

[3]   **4**   Propose how a problem can be solved using SMT with linear arithmetic and/or uninterpreted functions that was not covered in the lecture so far.

You do *not* have to implement it, but describe in detail how it could be done. In particular, describe all variables and constraints that are needed. (To exclude super easy problems let's say that the encoding should require at least 20 variables.)

[3]   ⋆ **5**   The schedule for the IT helpdesk staff of the Loanly Officers banking group is currently set up in a manual process using excel sheets; however, there are scenarios where the generated schedule violates some constraints. In order to deal with this issue, scheduling should be automated. The constraints are as follows:

(a) There are four types of 8-hour shifts: morning shift (06:00–14:00), afternoon shift (14:00–22:00), night shift (22:00–06:00), external shift (08:00–16:00).

(b) The staff consists of 9 operators and 5 supervisors who work morning, afternoon, and night in 8-hour shifts to provide a service that operates 24 hours a day and 7 days a week.

(c) The morning and afternoon shifts should include three employees (i.e., one supervisor and two operators).

(d) The night shift should include at least one person.

(e) Each employee is required to work 40 hours per week.

(f) The employee should not work on day shift after a night shift.

(g) Each employee must have at most one shift per day.

(h) Only operator staff can work at an external branch.

(i) If someone works an external shift in a certain week, he does so the entire week, i.e., from Monday to Friday, but in this case he has Saturday and Sunday off.