

SAT and SMT Solving | WS 2022 | LVA 703147

Exercises 10 | January 13, 2023

[2] 1 Give a bit blasting transformation for the signed comparisons $\geq_s$ and $>_s$, assuming that negative numbers are represented in two's complement. For example, $\mathbf{7}_4 >_s -\mathbf{2}_4$ is supposed to hold, and the constraints $\mathbf{x}_4 \geq_s -\mathbf{8}_4$ and $\mathbf{127}_8 \geq_s \mathbf{x}_8$ are valid.

[5] 2 Determine which of the following LLVM compiler optimizations correct, in the sense that the expressions before and after the arrow always correspond to the same values. Try to find a counterexample using an SMT encoding with bit vectors, for bit width 8 and 16.

```
%Op0 = lshr %X, C1
%r = udiv %Op0, C2
=>
%r = udiv %X, C2 << C1
```

```
Pre: isPowerOf2(%Power)
%s = shl %Power, %A
%Y = lshr %s, %B
%r = udiv %X, %Y
=>
%sub = sub %A, %B
%Y = shl %Power, %sub
%r = udiv %X, %Y
```

```
%na = sub 0, %a
%nb = sub 0, %b
%c = add %na, %nb
  =>
%ab = add %a, %b
%c = sub 0, %ab
```

- `Pre` indicates a precondition: the simplification is only applied if the precondition is satisfied. In the encoding, the precondition can therefore be asserted, because one is only interested in counterexamples which satisfy the precondition.
- `lshr` is a logical (unsigned) shift to the right, as provided by `bvlshr` in SMT-LIB.
- `udiv` is unsigned division, as provided by `bvudiv` in SMT-LIB.

[2] ⋆ 3 Give a bit blasting transformation for the left shift $\ll$ and the logical right shift $\gg_u$.

[3] 4 Bit hacks are popular in low-level programming. Check whether the following ones are correct.

(a) The website above claims that in order to compute the absolute value of an integer `x`, one can use either `(x + mask) ^ mask` or `(x ^ mask) - mask`, where `mask = x >> (NUM_BITS - 1)` and `NUM_BITS` is the bit width of `x`.

(b) This website claims that the expressions `(x + y) >> 1` and `((x ^ y) >> 1) + (x & y)` can be used to compute the average of two integers, i.e., $\lfloor (x+y)/2 \rfloor$.

Exercises marked with a ⋆ are optional. Solving them gives bonus points if you submit them before the course via OLAT or email.