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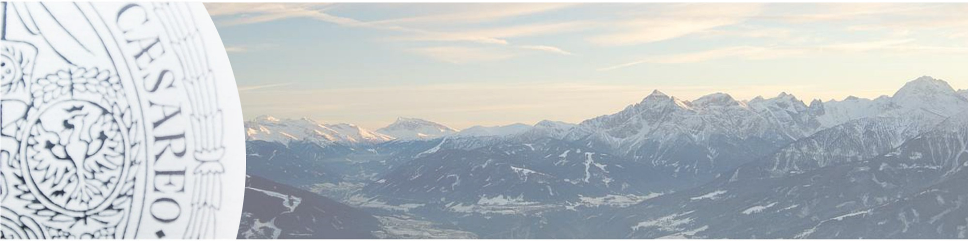

SAT and SMT Solving

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KRDB
Department of Computer Science
Free University of Bozen-Bolzano

lecture 10
WS 2022

Outline

- Summary of Last Week
- Bit Vectors

Satisfiability in Linear Integer Arithmetic

Definition (Cut)
given solution α to problem over $\mathbb{Q}^n$, cut is inequality $a_1x_1 + \cdots + a_nx_n \leqslant b$ which is not satisfied by α but by every $\mathbb{Z}^n$-solution

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Satisfiability in Linear Integer Arithmetic

Fact

for every LIA problem can compute bounds to get equisatisfiable bounded problem, so BranchAndBound terminates

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Gomory Cuts

Gomory Cuts: Assumptions

- DPLL(T) Simplex returned solution α and final tableau A such that

$$A\overline{x}_I = \overline{x}_D \qquad\qquad l_i \leqslant x_i \leqslant u_i$$

- for some $x_i \in D$ have $\alpha(x_i) \notin \mathbb{Z}$ and for all $x_j \in I$ value $\alpha(x_j)$ is l_j or u_j

Notation

- write $c = \alpha(x_i) - \lfloor \alpha(x_i) \rfloor$
- split independent variables I into $L = \{x_j \mid \alpha(x_j) = l_j\}$ and $U = \{x_j \mid \alpha(x_j) = u_j\}$
- $L^+ = \{ x_j \in L \mid \alpha(x_j) = l_j \text{ and } A_{ij} \geqslant 0 \}$ $\quad U^+ = \{ x_j \in U \mid \alpha(x_j) = u_j \text{ and } A_{ij} \geqslant 0 \}$
  $L^- = \{ x_j \in L \mid \alpha(x_j) = l_j \text{ and } A_{ij} < 0 \}$ $\quad U^- = \{ x_j \in U \mid \alpha(x_j) = u_j \text{ and } A_{ij} < 0 \}$

Lemma (Gomory Cut)

$$\sum_{x_j \in L^+} \frac{A_{ij}}{1-c}(x_j - l_j) - \sum_{x_j \in U^-} \frac{A_{ij}}{1-c}(u_j - x_j) - \sum_{x_j \in L^-} \frac{A_{ij}}{c}(x_j - l_j) + \sum_{x_j \in U^+} \frac{A_{ij}}{c}(u_j - x_j) \geqslant 1$$

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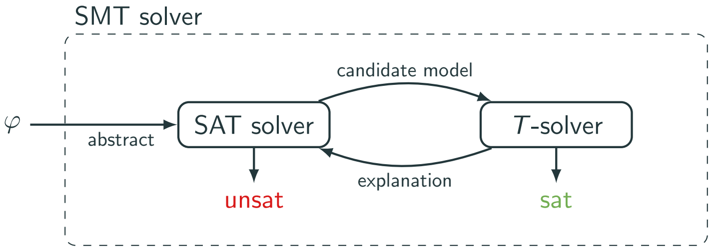
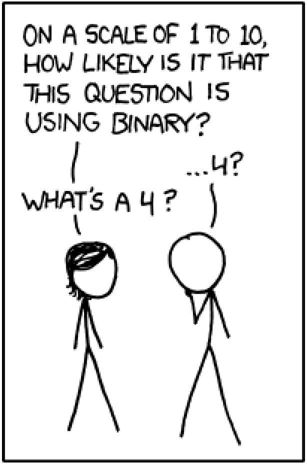
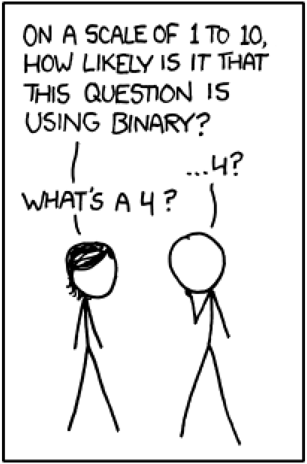
Theories in SMT Solving



Theory T	T-solving method	
equality logic	equality graphs	$\checkmark$
equality + uninterpreted functions (EUF)	congruence closure	$\checkmark$
linear real arithmetic (LRA)	Simplex	$\checkmark$
linear integer arithmetic (LIA)	Simplex + cuts + bounds	$\checkmark$
bitvectors (BV)	bit-blasting	
arrays (A)		


ON A SCALE OF 1 TO 10, HOW LIKELY IS IT THAT THIS QUESTION IS USING BINARY?
...4?
WHAT'S A 4?



Disclaimer

rest of lecture assumes brains in binary mode

Flashback to the Binary World

Binary representation

- k-bit representation of non-negative number n

$n \bmod 2^{k-1}$	$n \bmod 2^{k-2}$	$\cdots$	$n \bmod 2^1$	$n \bmod 2^0$

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- 5_4

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- …of negative number $-n$ is $(\sim n_k) + 1_k$ 2-complement

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- $\sim 5_4$: 1 0 1 0

Comparison operators

- $<_u$, $\leqslant_u$, ... considers operands as unsigned numbers

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Example

- $\boxed{0}\boxed{0}\boxed{1}\boxed{1} \quad <_u \quad \boxed{1}\boxed{1}\boxed{0}\boxed{1}$

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- 3_4 $\boxed{0}\boxed{0}\boxed{1}\boxed{1}$ $<_u$ $\boxed{1}\boxed{1}\boxed{0}\boxed{1}$ 13_4

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Binary operations and sign

- $+$, $-$, $\times$ work independently of whether operands are considered signed

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Example

$$
\begin{array}{r}
\boxed{0}\boxed{0}\boxed{1}\boxed{1} \\
+\ \boxed{1}\boxed{0}\boxed{1}\boxed{0} \\
\hline
\boxed{1}\boxed{1}\boxed{0}\boxed{1}
\end{array}
$$

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3_4		0	0	1	1	3_4
10_4	$+$	1	0	1	0	-6_4
13_4		1	1	0	1	-3_4

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- $+$, $-$, $\times$ work independently of whether operands are considered signed
- division and modulo depend on signedness: distinguish $\div_u$, $\%_u$ and $\div_s$, $\%_s$

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	1	0	1	0	10_4
$\div_u$	0	0	1	1	3_4
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13_4		1	1	0	1		0	0	1	1	3_4		1	1	1	0	-2_4

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Remark

- theories $\mathsf{BV}_{k_1}, \ldots \mathsf{BV}_{k_m}$ of different bit widths can be combined
- can also use binary $::$ for concatenation and unary $(\cdot)[i{:}j]$ to extract bits

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- variable $\mathbf{x}_k$ is list of length k of propositional variables $x_{k-1} \ldots x_2 x_1 x_0$
- valuation v assigns element in $\{\mathsf{T}, \mathsf{F}\}^k$ to variable $\mathbf{x}_k$,
  (usually written as binary number with k bits)

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  satisfiable: $v(\mathbf{x}_4) = \mathbf{4}_4$ and $v(\mathbf{y}_4) = \mathbf{3}_4$
- $\mathbf{x}_4 + \mathbf{2}_4 <_u \mathbf{x}_4$

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$\mathbf{x}[i{:}j]$ denotes $x_i \ldots x_j$
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satisfied by powers of 2 (and 0)

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bit blasting transformation $\mathbf{B}$ transforms BV formula into propositional formula:

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$\mathbf{B}_r$ transforms atom into propositional formula

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bit blasting $\mathbf{B}_t$ for term t returns (result u, side condition φ)

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for bit vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ set

- equality

$$\mathbf{B}_r(\mathbf{x}_{k+1} = \mathbf{y}_{k+1}) = (x_k \leftrightarrow y_k) \wedge \cdots \wedge (x_1 \leftrightarrow y_1) \wedge (x_0 \leftrightarrow y_0)$$

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- unsigned greater-than

$$\mathbf{B}(\mathbf{x}_k >_u \mathbf{y}_k) = \mathbf{B}(\mathbf{x}_k \geqslant \mathbf{y}_k) \wedge \mathbf{B}(\mathbf{x}_k \neq \mathbf{y}_k)$$

Definition (Bit Blasting: Bitwise Operations)

for bit vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ use fresh variable $\mathbf{z}_k$ and set

- bitwise and

$$\mathbf{B}_t(\mathbf{x}_k \mathbin{\&} \mathbf{y}_k) = (\mathbf{z}_k, \varphi) \qquad \varphi = \bigwedge_{i=0}^{k-1} z_i \leftrightarrow (x_i \wedge y_i)$$

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- bitwise negation

$$\mathbf{B}_t(-\mathbf{x}_k) = (\mathbf{z}_k, \varphi) \qquad \varphi = \bigwedge_{i=0}^{k-1} z_i \leftrightarrow \neg x_i$$

Definition (Bit Blasting: Concatenation, Extraction, If)

- concatenation

$$\mathbf{B}_t(\mathbf{x}_k :: \mathbf{y}_m) = (\mathbf{x}_k\mathbf{y}_m, \top)$$

for bit vectors $\mathbf{x}_k$ and $\mathbf{y}_m$

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- if-then-else

  $$\mathbf{B}_t(p \mathrel{?} \mathbf{x}_k : \mathbf{y}_k) = (\mathbf{z}_k, \varphi) \qquad \varphi = \bigwedge_{i=0}^{k-1} (p \to (z_i \leftrightarrow x_i)) \wedge (\neg p \to (z_i \leftrightarrow y_i))$$

  for formula p and bit vectors $\mathbf{x}_k$ and $\mathbf{y}_k$

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$$\bigwedge_{i=1}^{k-1} (c_i \leftrightarrow \mathrm{min2}(x_i, y_i, c_{i-1})) \wedge (s_i \leftrightarrow x_i \oplus y_i \oplus c_{i-1})$$

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- subtraction

$$\mathbf{B}_t(\mathbf{x}_k - \mathbf{y}_k) = \mathbf{B}_t(\mathbf{x}_k + (-\mathbf{y}_k))$$

Definition (Bit Blasting: Multiplication and Division)
for bit vectors $\mathbf{x}_k$ and $\mathbf{y}_k$ set

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$$\mathbf{B}_t(\mathbf{x}_k \div_u \mathbf{y}_k) = (\mathbf{q}_k, \varphi)$$

$$\varphi = \mathbf{B}(\mathbf{y}_k \neq \mathbf{0}_k \to (\mathbf{q}_k \times \mathbf{y}_k + \mathbf{r}_k = \mathbf{x}_k \wedge \mathbf{r}_k < \mathbf{y}_k \wedge \mathbf{q}_k < \mathbf{x}_k))$$

for fresh variables $\mathbf{q}_k$ and $\mathbf{r}_k$

Example (SMT-LIB 2 for BV)

$(\mathbf{a}_4 + \mathbf{b}_4 <_u \mathbf{b}_4) \wedge (\mathbf{a}_4 \neq \mathbf{10}_4) \wedge (\mathbf{a}_4 \,\&\, \mathbf{b}_4 = \mathbf{8}_4)$ is expressed as

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(declare-const a (_ BitVec 4))
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(assert (= (bvand a b) #b1000))
(check-sat)
```

Bit vectors in SMT-LIB 2

- `(_ BitVec k)` is sort of bitvectors of length k
- `#xa` is constant in hexadecimal
- `#b1000` is constant in binary
- `bvadd`, `bvsub`, `bvmul` are arithmetic operations,
  `bvudiv` and `bvsdiv` are unsigned and signed division
- `bvult` and `bvule` are unsigned, `bvslt` and `bvsle` are signed $<$ and $\leqslant$
- `bvshl`, `bvlshr`, `bvashr` are shifts
- `bvand`, `bvor` are bitwise logical operations

Bit Vectors in python/z3

```python
from z3 import *
x = BitVec("x", 8)
y = BitVec("y", 8)
zero = BitVecVal(0, 8)
one = BitVecVal(1, 8)
r = y ^ ((x ^ y) & (zero -(If(x < y, one, zero))))
m = If(x<y, x, y)
solve(r != m) # shorthand for checking single formula
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- for valuations, solver returns integers by default

Application 1: Verifying Compiler Optimizations

LLVM

- open-source umbrella project: set of reusable toolchain components: libraries, assemblers, compilers, debuggers, ...

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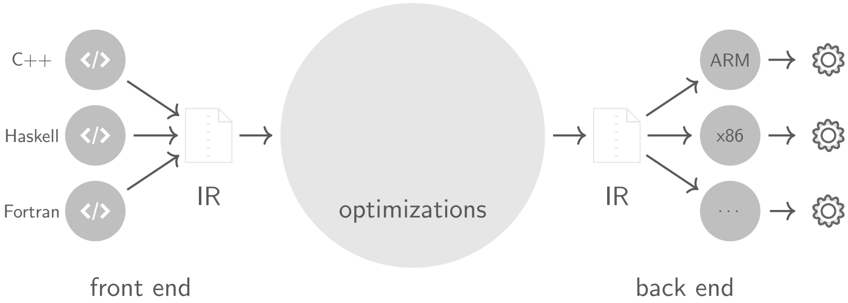

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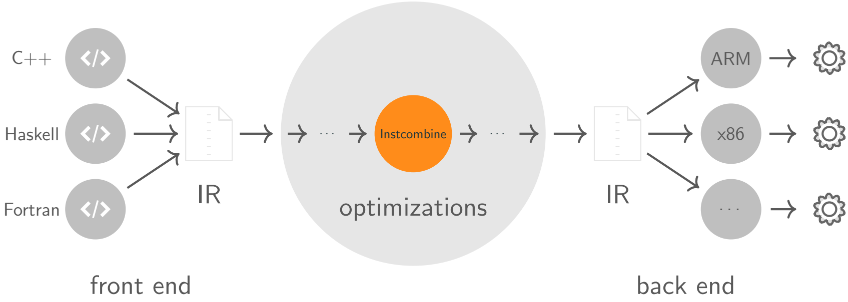
- compilation toolchain includes peephole optimizations in Instcombine pass



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Instcombine Pass

- over 1000 algebraic simplifications of expressions
  - transform multiplies with constant power-of-two argument into shifts
  - bitwise operators with constant operands are always grouped so that shifts are performed first, then ors, then ands, then xors
  - changing bitwidth of variables
  - ...

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Example

```c
int foo(int z) {
int x = 4 * (z | 1001);
return -256 & x;
}
```

```llvm
define i32 @foo(i32) #0 {
  %2 = or i32 %0, 1001
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```llvm
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  %2 = shl i32 %0, 2
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  - ...
- code is community maintained
- sometimes optimizations have errors—and compiler bugs are critical

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Alive Project

- represent Instcombine optimizations in domain-specific language, e.g.

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Name: PR20186
%a = sdiv %X, C
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- check correctness by means of SMT encoding

```
(declare-const x (_ BitVec 32))
(declare-const c (_ BitVec 32))
(declare-const before (_ BitVec 32))
(declare-const after (_ BitVec 32))
(assert (= before (bvsub #x00000000 (bvsdiv x c))))
(assert (= after (bvsdiv x (bvneg c))))
(assert (not (= before after)))
(assert (not (= c #x00000000)))
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- wrong for `c = x = #x80000000`

Same in python/z3

```
from z3 import *
x = BitVec('x', 32) # create variable named x with 32 bits
c = BitVec('c', 32)

before = BitVecVal(0, 32) - (x / c)
after = x / - c

solver = Solver()
solver.add(c != BitVecVal(0, 32)) # exclude case where c=0
solver.add(after != before)

result = solver.check()
if result == z3.sat:
  m = solver.model()
  print m[x], m[c] # 2147483648 2147483648
  print m.eval(before), m.eval(after) # 4294967295 1
```

Application 2: Detecting Nontermination in Programs

```
int bsearch(int a[], int k, unsigned int lo, unsigned int hi) {
  unsigned int mid;
  while (lo < hi) {
    mid = (lo + hi)/2;
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      lo = mid + 1;
    else if (a[mid] > k)
      hi = mid - 1;
    else
      return mid;
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  return -1;
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- (former) implementation of binary search in Java library

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- (former) implementation of binary search in Java library
- loops for inputs `lo=1` and `hi=UINT_MAX` if `a[0] < k`.
- SMT encoding can find values such that parameters stay the same in recursive call

Bibliography

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