$\boxed{1}$ Given the formula $\phi$:

$$(1 \vee \neg 2) \wedge (\neg 1 \vee 3) \wedge (4 \vee \neg 3) \wedge (\neg 4 \vee \neg 1) \wedge (\neg 4 \vee \neg 3 \vee 1) \wedge (2 \vee 3 \vee 1) \wedge (\neg 4 \vee 2 \vee 5)$$

the following DPLL inference sequence shows its unsatisfiability:

$$\begin{aligned}
\parallel \phi &\Longrightarrow 1^d \parallel \phi && \text{decide}\\
&\Longrightarrow 1^d\, 3 \parallel \phi && \text{unit propagate}\\
&\Longrightarrow 1^d\, 3\, 4 \parallel \phi && \text{unit propagate}\\
&\Longrightarrow \overline{1} \parallel \phi && \text{backtrack}\\
&\Longrightarrow \overline{1}\, \overline{2} \parallel \phi && \text{unit propagate}\\
&\Longrightarrow \overline{1}\, \overline{2}\, 3 \parallel \phi && \text{unit propagate}\\
&\Longrightarrow \overline{1}\, \overline{2}\, 3\, \overline{4} \parallel \phi && \text{unit propagate}\\
&\Longrightarrow \mathsf{FailState} && \text{fail}
\end{aligned}$$

$\boxed{2}$ We consider the following additional variables:

$a_0\colon \neg(p \vee \neg(q \vee r)) \wedge (q \vee r)$ $\qquad a_1\colon q \vee r$

$a_2\colon \neg(p \vee \neg(q \vee r))$ $\qquad a_3\colon p \vee \neg(q \vee r)$

$a_4\colon \neg(q \vee r)$

(a) Writing $\overline{a_i}$ for $\neg a_i$, Tseitin's transformation leads to a formula with 14 clauses:

$$\begin{aligned}
\phi &\approx a_0 \wedge (a_0 \leftrightarrow a_2 \wedge a_1) \wedge (a_1 \leftrightarrow q \vee r) \wedge (a_2 \leftrightarrow \overline{a_3}) \wedge (a_3 \leftrightarrow p \vee a_4) \wedge (a_4 \leftrightarrow \overline{a_1})\\
&\equiv a_0 \wedge (\overline{a_0} \vee a_2) \wedge (\overline{a_0} \vee a_1) \wedge (\overline{a_2} \vee \overline{a_1} \vee a_0) \wedge\\
&\quad (\overline{a_1} \vee q \vee r) \wedge (\overline{q} \vee a_1) \wedge (\overline{r} \vee a_1) \wedge\\
&\quad (\overline{a_2} \vee \overline{a_3}) \wedge (a_2 \vee a_3) \wedge\\
&\quad (\overline{a_3} \vee p \vee a_4) \wedge (\overline{p} \vee a_3) \wedge (\overline{a_4} \vee a_3) \wedge\\
&\quad (\overline{a_4} \vee \overline{a_1}) \wedge (a_4 \vee a_1)
\end{aligned}$$

(b) The transformation by Plaisted and Greenbaum produces a formula $\psi$ with 10 clauses:

$$\begin{aligned}
\phi &\approx a_0 \wedge (a_0 \rightarrow a_2 \wedge a_1) \wedge (a_1 \rightarrow q \vee r) \wedge (a_2 \rightarrow \overline{a_3}) \wedge (a_3 \leftarrow p \vee a_4) \wedge (a_4 \leftarrow \overline{a_1})\\
&\equiv a_0 \wedge (\overline{a_0} \vee a_2) \wedge (\overline{a_0} \vee a_1) \wedge\\
&\quad (\overline{a_1} \vee q \vee r) \wedge\\
&\quad (\overline{a_2} \vee \overline{a_3}) \wedge\\
&\quad (\overline{p} \vee a_3) \wedge (\overline{a_4} \vee a_3) \wedge\\
&\quad (a_4 \vee a_1)
\end{aligned}$$

The following DPLL inference sequence shows satisfiability of $\psi$:

$$
\begin{aligned}
\parallel \psi &\Longrightarrow a_0 \parallel \psi && \textsf{unit propagate}\\
&\Longrightarrow a_0\, a_1 \parallel \psi && \textsf{unit propagate}\\
&\Longrightarrow a_0\, a_1\, a_2 \parallel \psi && \textsf{unit propagate}\\
&\Longrightarrow a_0\, a_1\, a_2\, \overline{a_3} \parallel \psi && \textsf{unit propagate}\\
&\Longrightarrow a_0\, a_1\, a_2\, \overline{a_3}\, \overline{p} \parallel \psi && \textsf{unit propagate}\\
&\Longrightarrow a_0\, a_1\, a_2\, \overline{a_3}\, \overline{p}\, \overline{a_4} \parallel \psi && \textsf{unit propagate or pure literal}\\
&\Longrightarrow a_0\, a_1\, a_2\, \overline{a_3}\, \overline{p}\, \overline{a_4}\, q^d \parallel \psi && \textsf{decide}\\
&\Longrightarrow a_0\, a_1\, a_2\, \overline{a_3}\, \overline{p}\, \overline{a_4}\, q^d\, r^d \parallel \psi && \textsf{decide}
\end{aligned}
$$

(The last step is only necessary if a total assignment is desired.)

$\boxed{3}$ See `minesweeper.py`.

$\boxed{4}$ To model the problem, we have the set of routers $R = \{\mathtt{r1}, \mathtt{r2}, \ldots, \mathtt{r7}\}$ and the set of destinations $D = \{d_1, d_2, d_3, d_4\}$ for $d_1 = \textsf{10.91.110.}*$, $d_2 = \textsf{10.91.120.}*$, $d_3 = \textsf{10.91.130.}*$, $d_4 = \textsf{10.91.140.}*$. We can use $7 * 4 * 7 = 196$ variables $x_{i,d,j}$ for all combinations of $\mathtt{r}_i, \mathtt{r}_j \in R$ and $d \in D$, with the intended meaning that $x_{i,d,j}$ is assigned true iff $\mathtt{r}_i$ sends packages with destination $d$ to $\mathtt{r}_j$, or $i = j$ and $d$ is in the VLAN of $\mathtt{r}_i$.

First, we must ensure that every router must forward packages:

$$\bigwedge_{i=1}^{7} \bigwedge_{d\in D} \bigvee_{j=1}^{7} x_{i,d,j}$$

Next, the rules can be encoded as follows:

(C1) No router should send a package to itself, so forbid $x_{i,d,i}$ unless $d$ is in the VLAN of $\mathtt{r}_i$.

$$\bigwedge_{i\in\{3,4,7\}} \bigwedge_{d\in D} \neg x_{i,d,i}$$

$$\left(\bigwedge_{d\in D\setminus\{d_1\}} \neg x_{1,d,1}\right) \wedge \left(\bigwedge_{d\in D\setminus\{d_2\}} \neg x_{2,d,2}\right) \wedge \left(\bigwedge_{d\in D\setminus\{d_3\}} \neg x_{5,d,5}\right) \wedge \left(\bigwedge_{d\in D\setminus\{d_4\}} \neg x_{6,d,6}\right)$$

(C2) Traffic directed to an address in a different cluster is forwarded to a router above (if such a router exists). From the picture, one can see that e.g. the routers above $\mathtt{r}_1$ are $\mathtt{r}_7$ and $\mathtt{r}_4$, and the destinations in a different cluster are $d_3$ and $d_4$. Thus we get:

$$\bigwedge_{d\in\{d_3,d_4\}} (x_{1,d,4} \vee x_{1,d,7}) \wedge \bigwedge_{d\in\{d_3,d_4\}} (x_{2,d,4} \vee x_{2,d,7}) \wedge \bigwedge_{d\in\{d_1,d_2\}} (x_{3,d,4} \vee x_{3,d,3}) \wedge \bigwedge_{d\in\{d_1,d_2\}} (x_{4,d,4} \vee x_{4,d,3})$$

(C3) Two routers in the same cluster and on the same level (i.e. not one above the other) route to the same destination. This concerns the router pairs $(\mathtt{r}_1, \mathtt{r}_2)$ and $(\mathtt{r}_5, \mathtt{r}_6)$.

$$\bigwedge_{d\in D} \bigwedge_{j=1}^{7} (x_{1,d,j} \leftrightarrow x_{2,d,j}) \wedge \bigwedge_{d\in D} \bigwedge_{j=1}^{7} (x_{5,d,j} \leftrightarrow x_{6,d,j})$$

The current routing tables set the following variables to true:

$$
\begin{aligned}
&(x_{1,d_2,2} \wedge x_{1,d_2,7} \wedge x_{1,d_3,4} \wedge x_{1,d_4,7}) \wedge\\
&(x_{2,d_1,1} \wedge x_{2,d_3,5} \wedge x_{2,d_4,5} \wedge x_{2,d_4,6}) \wedge\\
&(x_{5,d_1,7} \wedge x_{5,d_2,4} \wedge x_{5,d_3,4} \wedge x_{5,d_4,3} \wedge x_{5,d_4,6})
\end{aligned}
$$

and all other variables $x_{1,d,j}$, $x_{2,d,j}$, and $x_{5,d,j}$ to false. One problem is thus that $x_{1,d_3,4} \leftrightarrow x_{2,d_3,4}$ is not satisfied.