SAT and SMT Solving | WS 2022 | LVA 703147

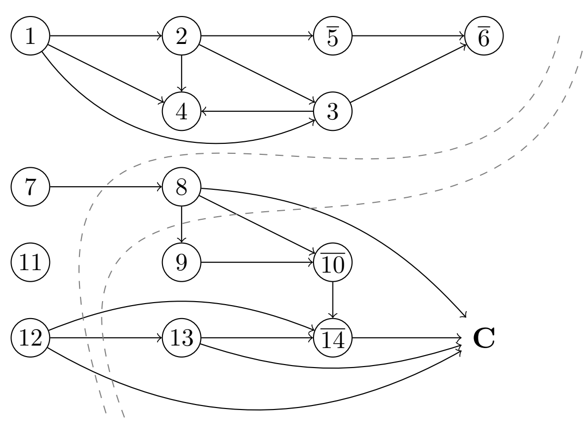
Solutions 2 | October 28, 2022

1 We consider the formula φ:

$$(\neg 1 \vee 2) \wedge (\neg 1 \vee \neg 2 \vee 3) \wedge (\neg 1 \vee \neg 2 \vee \neg 3 \vee 4) \wedge (\neg 2 \vee \neg 5) \wedge (\neg 3 \vee 5 \vee \neg 6) \wedge$$
$$(\neg 7 \vee 8) \wedge (\neg 8 \vee 9) \wedge (\neg 8 \vee \neg 9 \vee \neg 10) \wedge (\neg 12 \vee 13) \wedge (10 \vee \neg 12 \vee \neg 13 \vee \neg 14) \wedge$$
$$(\neg 8 \vee \neg 12 \vee \neg 13 \vee 14)$$

and a DPLL inference sequence reaching state $1^d\ 2\ 3\ 4\ \overline{5}\ \overline{6}\ 7^d\ 8\ 9\ \overline{10}\ 11^d\ 12^d\ 13\ \overline{14}$.

(a) The implication graph looks as follows:



The single unique implication point is node 12. The two indicated cuts lead to the implied clause $\neg 7 \vee \neg 12$ and $\neg 8 \vee \neg 12$. These are the smallest implied clauses obtained from cuts in this graph.

(b) The conflict clause is $\neg 8 \vee \neg 12 \vee \neg 13 \vee 14$, its literal whose complement was assigned last is 14. We thus resolve

$$\frac{\neg 8 \vee \neg 12 \vee \neg 13 \vee 14 \qquad 10 \vee \neg 12 \vee \neg 13 \vee \neg 14}{\neg 8 \vee 10 \vee \neg 12 \vee \neg 13}$$

The literal in the resulting clause whose complement was assigned last is 13. We hence get

$$\frac{\neg 8 \vee 10 \vee \neg 12 \vee \neg 13 \qquad \neg 12 \vee 13}{\neg 8 \vee 10 \vee \neg 12}$$

The last-assigned literal is now 12, a decision literal. But we can keep resolving with the clause that led to the assignment of the last non-decision literal, namely 10:

$$\frac{\neg 8 \vee 10 \vee \neg 12 \qquad \neg 8 \vee \neg 9 \vee \neg 10}{\neg 8 \vee \neg 9 \vee \neg 12}$$

Now the last assigned non-decision literal is 9, so obtain a next resolution step

$$\frac{\neg 8 \lor \neg 9 \lor \neg 12 \qquad \neg 8 \lor 9}{\neg 8 \lor \neg 12}$$

We proceed with a resolution step eliminating 8:

$$\frac{\neg 8 \lor \neg 12 \qquad \neg 7 \lor 8}{\neg 7 \lor \neg 12}$$

At this point there are only decision literals left, so no further steps are possible.

$\boxed{2}$ The DPLL inference sequence is as follows:

$$\begin{array}{rll}
 & \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \boxed{\overline{3}} \lor \boxed{\overline{4}} \lor 8,\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}} & \\
\Longrightarrow 1^d & \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \boxed{\overline{3}} \lor \boxed{\overline{4}} \lor 8,\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}} & \textsf{decide} \\
\Longrightarrow 1^d\, 7 & \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \boxed{\overline{3}} \lor \boxed{\overline{4}} \lor 8,\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}} & \textsf{up} \\
\Longrightarrow 1^d\, 7\, 3^d & \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \overline{3} \lor \boxed{\overline{4}} \lor \boxed{8},\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}} & \textsf{decide} \\
\Longrightarrow 1^d\, 7\, 3^d\, 4^d & \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \overline{3} \lor \boxed{\overline{4}} \lor \boxed{8},\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}} & \textsf{decide} \\
\Longrightarrow 1^d\, 7\, 3^d\, 4^d\, 8 & \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \overline{3} \lor \boxed{\overline{4}} \lor \boxed{8},\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}} & \textsf{up} \\
\Longrightarrow 1^d\, 7\, 3^d\, 4^d\, 8\, 5^d & \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \overline{3} \lor \boxed{\overline{4}} \lor \boxed{8},\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}} & \textsf{decide} \\
\Longrightarrow 1^d\, 7\, 3^d\, 4^d\, 8\, 5^d\, 6^d & \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \overline{3} \lor \boxed{\overline{4}} \lor \boxed{8},\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}} & \textsf{decide}
\end{array}$$

with up abbreviating unit propagate, and where the framed literals are watched. We can then do a backjump step using backjump clause $\overline{1} \lor \overline{3} \lor \overline{8}$ to

$$1^d\, 7\, 3^d\, \overline{8} \ \| \ \boxed{1} \lor \boxed{\overline{2}},\ \boxed{\overline{1}} \lor \boxed{7},\ \boxed{3} \lor \boxed{4},\ \overline{3} \lor \boxed{\overline{4}} \lor \boxed{8},\ \boxed{4} \lor \boxed{5} \lor 6,\ \boxed{\overline{8}} \lor \boxed{\overline{3}}$$

which could be simulated in three backtrack steps.

$\boxed{3}$ See the file gardening.py.