1 The following constraints and pictures match:

(1)-(B) The solution space is the triangle in the picture. The problem is unsatisfiable over $\mathbb{Z}$ but satisfiable over $\mathbb{Q}$, and bounded.

(2)-(C) The solution space between the pink and the yellow line. The problem is unsatisfiable over $\mathbb{Z}$ but satisfiable over $\mathbb{Q}$, and unbounded.

(3)-(D) The solution space is the area containing coordinate $(1, 0)$. The problem is satisfiable over $\mathbb{Z}$ and $\mathbb{Q}$, and unbounded.

(4)-(E) The problem is unsatisfiable over $\mathbb{Z}$ and $\mathbb{Q}$, and by definition bounded.

(5)-(A) The solution space is the triangle in the picture. The problem is bounded, and satisfiable over both $\mathbb{Q}$ and $\mathbb{Z}$, e.g. by $x = 2$, $y = 2$.

(6)-(F) The solution space is the triangle in the picture. The problem is bounded, and satisfiable over both $\mathbb{Q}$ and $\mathbb{Z}$, e.g. by $x = 1$, $y = 2$.

3 (a) From the initial tableau (left) a solution to the problem over $\mathbb{R}^2$ can be obtained with the Simplex algorithm, together with a final tableau (right):

$$\begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{pmatrix} -3 & 2 \\ 0 & 1 \\ -2 & -1 \\ 3 & -1 \end{pmatrix} \end{array} \begin{array}{l} s_1 \leq 1 \\ s_2 \leq 4 \\ s_3 \leq -5 \\ s_4 \leq 7 \end{array} \qquad \begin{array}{c} \\ s_3 \\ x \\ y \\ s_4 \end{array} \begin{array}{c} \begin{array}{cc} s_2 & s_1 \end{array} \\ \begin{pmatrix} -2\frac{1}{3} & 1\frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \\ 1 & 0 \\ 1 & -1 \end{pmatrix} \end{array} \begin{array}{l} x = 2\frac{1}{3} \\ y = 4 \\ s_1 = 1 \\ s_2 = 4 \\ s_3 = -8\frac{2}{3} \\ s_4 = 3 \end{array}$$

We pick the variable x which is assigned $2\frac{1}{3} \notin Z$, so $c = \frac{1}{3}$. Since there are only upper bounds, the set of nonbasic variables L is empty and the Gomory cut inequality simplifies to the following form:

$$-\sum_{j \in U^-} \frac{A_{ij}}{1-c}(u_j - x_j) + \sum_{j \in U^+} \frac{A_{ij}}{c}(u_j - x_j) \geq 1 \qquad (\star)$$

Due to the coefficients in the tableau we have $U^- = \{s_1\}$ and $U^+ = \{s_2\}$. So $(\star)$ amounts to

$$-(-\frac{1}{3}/\frac{2}{3})(1 - s_1) + (\frac{2}{3}/\frac{1}{3})(4 - s_2) \geq 1$$

or equivalently

$$-\frac{1}{2}s_1 - 2s_2 \geq -7\frac{1}{2}$$

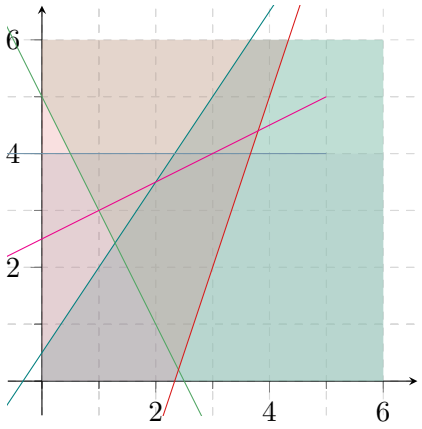
Using the equations $s_1 = -3x + 2y$ and $s_2 = y$, this is equivalent to

$$-\frac{1}{2}(-3x + 2y) - 2y \geq -7\frac{1}{2}$$

After some simplifications, we get

$$\frac{5}{2} + \frac{1}{2}x \geq y$$

This cut corresponds to the magenta line in the picture, cutting off the last solution:



3 See the file `loanly_officers.py`.