universität innsbruck

Automata and Logic	WS 2023/2024	LVA 703302
1st EXAM		January 29, 2024

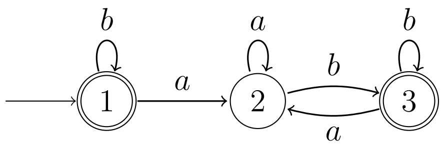
This exam consists of **four** exercises. The available points for each item are written in the margin. ***Explain your answers!***

1 Determine whether the following sets over $\Sigma = \{a, b\}$ are regular or not. Prove your answers.

⟨10⟩ (a) $\{(a^n b)^m \mid n > m > 0\}$

⟨10⟩ (b) $\{x^n \mid \#a(x) \neq \#b(x) \text{ and } n > 0\}$

⟨10⟩ **2** (a) Construct a WSMO formula φ such that $L(\varphi) = L(M)$ for the following DFA M:



⟨15⟩ (b) Consider the WSMO formula $\varphi = \exists X.\, \exists y.\, y < x \wedge X(y)$. Give automata or regular expressions for the atomic subformulas and explain the operations needed to obtain the regularity of $L_a(\varphi)$.

3 Consider the LTL formula $\varphi = \neg(\neg a \,\mathsf{U}\, \mathsf{X}\, b) \wedge \neg \mathsf{G}\, b$.

⟨10⟩ (a) Transform φ into negation normal form ψ.

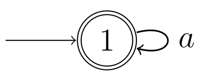
⟨15⟩ (b) Use the construction from the lecture to compute the alternating Büchi automaton A_ψ.

⟨10⟩ (c) Which of the following traces are accepted by A_ψ?

i. $\{a\}\{b\}^\omega$ ii. $(\{a, b\}\varnothing\{a, b\})^\omega$ iii. $\varnothing^\omega$

⟨20⟩ **4** Determine whether the following statements are true or false. (Providing explanation is optional.) Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

1. There exists a non-regular set that is accepted by an AFA.
2. MONA is a tool that implements algorithms for Büchi automata.
3. The class of ω-regular sets is effectively closed under intersection.
4. In a completely defined tree automaton, all states are productive.
5. Every Büchi automaton can be effectively transformed into a DBA.
6. Every regular set is accepted by a DFA having exactly one final state.
7. The automaton A_φ for the Presburger formula $\varphi\colon 3x - 2y = 1$ has at most 14 states.
8. The MSO formula $(\forall x.\, x = 0 \rightarrow X(x)) \wedge \neg(\forall x.\, X(x) \rightarrow \exists y.\, x < y \wedge X(y))$ is satisfiable.
9. For the WMSO formula $\varphi = \forall y.\, y < x \rightarrow X(y)$ we have
$$L_a(\varphi) = \binom{0}{0}^* \left[\binom{1}{0} + \binom{1}{1}\right] \left[\binom{0}{0} + \binom{0}{1}\right]^*$$
10. The equivalence relation $\sim_M$ for the NBA M



over the alphabet $\Sigma = \{a, b\}$ has 2 equivalence classes.