Automata and Logic — WS 2023/2024 — LVA 703302

2nd EXAM — February 23, 2024

This exam consists of **five** exercises. The available points for each item are written in the margin. ***Explain your answers!***

⟨10⟩ **1** Consider the alphabet $\Sigma = \{a, b\}$. Give a set $A \subseteq \Sigma^*$ and a homomorphism $h \colon \Sigma^* \to \Sigma^*$ such $h(A)$ and $h^{-1}(A)$ are regular but A is not.

⟨15⟩ **2** Construct an AFA with less than 100 states for the set $\{x \in \{a\}^* \mid |x| = 2 \mod 105\}$. (Hint: consider the prime factorization of 105.)

⟨15⟩ **3** Construct a WMSO formula for the set $\{x \in \{a, b\}^* \mid |x| \text{ is even}\}$.

4 Consider the Presburger arithmetic formula $\varphi = \exists\, x.\, \exists\, z.\, 2x - 3y + 2z = 1$.

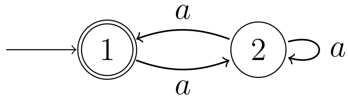
⟨10⟩ (a) Which of the following strings belong to $L(\varphi)$?

i. 0011 ii. 1001 iii. 1000

⟨15⟩ (b) Construct a finite automaton that accepts $L(2x - 3y + 2z = 1)$.

⟨15⟩ (c) Construct a finite automaton that accepts $L(\varphi)$.

⟨20⟩ **5** Determine whether the following statements are true or false. (Providing explanation is optional.) Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

1. The set $\{xx \mid x \in \{a, b\}^*\}$ is WMSO definable.
2. The regular expressions $\boldsymbol{\epsilon}^*$ and $\boldsymbol{\varnothing}^*$ are equivalent.
3. The string a^ω belongs to the ω-iteration of $\{\epsilon, aa\}$.
4. The WMSO formula $\varphi = \forall\, X.\, (\exists\, x.\, X(x) \to \exists\, y.\, \neg\, X(y))$ is valid.
5. The Hamming distance between the strings 010101 and 11011 is 5.
6. The sets L_{11} and L^{f}_{11} coincide for the NBA 

7. Every regular set is accepted by an NFA having exactly one start state.
8. According to Arden's Lemma, $X = aX + bY + c$ implies $X = (bY + c)^* a$.
9. If $\equiv$ is a Myhill–Nerode relation on Σ^* then $x \equiv y$ implies $ax \equiv ay$ for all $a \in \Sigma$.
10. The string

$$\begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

is 2-admissable.