universität innsbruck

Computability Theory	WS 2023	LVA 703317
Week 1		October 9, 2023

Exercises

⟨3⟩ 1. Prove that the following functions are primitive recursive:

(a) $f(x) = \begin{cases} x \div 2 & \text{if } x \text{ is even} \\ 3x+1 & \text{if } x \text{ is odd} \end{cases}$

(b) The function $g(x)$ that returns the natural number y such that $y \leqslant \sqrt{x} < y + 1$.

(c) $\mathsf{gcd}(x, y)$

⟨2⟩ 2. Consider the function $\pi'(x, y) = ((x + y)^2 + 3x + y) \div 2$.

(a) Prove that π' is a bijection between $\mathbb{N}^2$ and $\mathbb{N}$

(b) Define corresponding primitive recursive extraction functions π'_1 and π'_2.

⟨2⟩ 3. Let c be an arbitrary natural number.

(a) Let $f \colon \mathbb{N}^{n+1} \to \mathbb{N}$ be a primitive recursive function with $n > 0$. Prove that the function $g \colon \mathbb{N}^n \to \mathbb{N}$ defined by

$$g(x_1, \dots, x_n) = f(x_1, \dots, x_n, c)$$

is primitive recursive.

(b) Let $h \colon \mathbb{N}^2 \to \mathbb{N}$ be primitive recursive. Prove that the function $f \colon \mathbb{N} \to \mathbb{N}$ defined by the recursive equations

$$\begin{aligned} f(0) &= c \\ f(x+1) &= h(f(x), x) \end{aligned}$$

is primitive recursive.