universität innsbruck

Computability Theory	WS 2023	LVA 703317
Week 2		October 16, 2023

Exercises

⟨3⟩ 1. Let c be an arbitrary natural number.

   (a) Prove that $\mathsf{ack}(c, x)$ is a primitive recursive function.

   (b) Is $\mathsf{ack}(x, c)$ primitive recursive?

⟨2⟩ 2. Which primitive recursive function has index $\langle 4, \langle 1\rangle, \langle 3, \langle 1\rangle, \langle 2, 3, 1\rangle\rangle\rangle$?

⟨2⟩ 3. Which number is the encoding of the computation of $\mathsf{ack}(2, 1) = 5$?

Bonus Exercise

⟨2⟩ 4. Consider the primitive recursive function $\beta\colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ defined as

$$\beta(a, i) \;=\; \pi_1(a) \;\mathsf{mod}\; \big(1 + (i + 1)\pi_2(a)\big)$$

Prove that for every finite sequence $a_0, \dots, a_n$ of natural numbers there exists a natural number a such that $\beta(a, i) = a_i$ for all $0 \leqslant i \leqslant n$.