

Computability Theory

WS 2023

LVA 703317

```
Week 3
```

October 23, 2023

Exercises

 $\langle 1 \rangle$ 1. Which function is computed by the following LOOP program P(x, y; z)?

```
 \begin{array}{l} z := x; \\ {\sf LOOP} \; y \; {\sf DO} \\ v := 0; \\ w := 0; \\ {\sf LOOP} \; z \; {\sf DO} \\ v := w; \\ w++ \\ {\sf OD}; \\ z := v \\ {\sf OD}; \\ z++ \end{array}
```

- $\langle 2 \rangle$ 2. Complete the proof on slide 14 that every LOOP computable function is primitive recursive.
- $\langle 2 \rangle$ 3. Construct LOOP programs for the following functions:
 - (a) x y
 - (b) $\lfloor \frac{x}{2} \rfloor$
- $\langle 2 \rangle$ 4. Prove parts (3) and (4) of the lemma on slide 29.

Bonus Exercise

- $\langle 3 \rangle$ 5. Determine the level in the Grzegorczyk hierarchy of the Fibonacci function.
- $\langle 1 \rangle$ 6. Does the inequality $2_2(x) \times 2_c(x) \leq 2_{c+3}(x)$ hold for all $x, c \ge 0$?