Computability Theory	WS 2023	LVA 703317
Week 5		November 6, 2023

Exercises

⟨2⟩ 1. Complete the proof of Kleene's normal form theorem.

⟨2⟩ 2. Define a primitive recursive function arity such that $\mathsf{arity}(x) = n$ if x is the index of an n-ary partial recursive function and 0 otherwise.

⟨1⟩ 3. Define a *universal* partial recursive function $\varphi\colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ which generates all partial recursive functions of any arity, in the sense that for every n-ary partial recursive function f there exists a natural number e such that $f(x_1, \dots, x_n) \simeq \varphi(e, \langle x_1, \dots, x_n \rangle)$.

⟨2⟩ 4. Prove that there is no primitive recursive predicate *recursive* that tests whether its argument is the index of a total recursive function.

Bonus Exercise

⟨2⟩ 5. Prove that for every two recursive functions $f, g\colon \mathbb{N}^2 \to \mathbb{N}$ there exist $a, b \in \mathbb{N}$ such that $\varphi_{f(a,b)} \simeq \varphi_a$ and $\varphi_{g(a,b)} \simeq \varphi_b$.