

## Exercises

- (3) 1. Prove the lemma on [slide 11](#).
- (2) 2. Prove that the set of natural numbers which are not powers of two is diophantine.
- (2) 3. Prove that diophantine sets are closed under union and intersection.

## Bonus Exercise

4. Let us write  $(\nu i)(g(i, \vec{y}) = 0)$  for the smallest  $i$  such that  $g(i, \vec{y}) = 0$  and, for all  $j < i$ , either  $g(j, \vec{y}) \uparrow$  or  $g(j, \vec{y}) > 0$ .
  - (1) (a) Construct a partial recursive function  $\varphi$  such that  $(\mu i)(\varphi(i, \vec{y}) = 0)$  and  $(\nu i)(\varphi(i, \vec{y}) = 0)$  are different functions.
  - (1) (b) Show that the class of partial recursive functions is not closed under the  $\nu$  operator.
  - (1) (c) In light of the result of part (b), why don't we replace  $\mu$  by  $\nu$  in the definition of partial recursive functions?