universität innsbruck

Computability Theory	WS 2023	LVA 703317
Week 11		January 8, 2024

Exercises

⟨2⟩ 1. Compute the principal type of C.

⟨2⟩ 2. Complete the proof of the lemma on slide 11.

⟨1⟩ 3. (a) Prove the first lemma on slide 26.

⟨2⟩ (b) Prove the second lemma on slide 26.

Bonus Exercise

4. Consider the functions $h_n \colon \mathbb{N} \to \mathbb{N}$ for $n \geqslant 0$ defined by

$$h_0(x) = x + 1 \qquad\qquad h_{n+1}(x) = h_n^{(x)}(x)$$

⟨1⟩ (a) Prove that $h_2(x) = x \times 2^x$.

⟨1⟩ (b) Prove that every h_n is primitive recursive.

⟨2⟩ (c) Prove that for every primitive recursive function f there exists a $c \geqslant 0$ such that

$$f(x_1, \ldots, x_n) < h_c(\mathsf{max}\{x_1, \ldots, x_n\})$$

for *almost every* $x_1, \ldots, x_n \in \mathbb{N}$.

⟨1⟩ (d) Is the function $g(x) = h_x(x)$ primitive recursive?