WS 2024/2025

Advanced Functional Programming

Week 8 – Backtracking during Parsing, Applicative Functors, Monad Transformers

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Last Week

- context free grammars
- parser combinators, example: Parsec library
  - several primitives to read a single char, e.g., `char`, `anyOf`, `noneOf`, `space`, `satisfy`, `eof`
  - combinators to combine parsers, e.g., `many`, `many1`, `sepBy`, `endBy`
  - `p1 <|> p2` and `try p1` are used for non-determinism and back-tracking
    - if `p1` succeeds, then `p1 <|> p2` and `try p1` succeed
    - if `p1` fails after consuming some input, then `p1 <|> p2` fails
    - if `p1` fails without consuming input, then `p1 <|> p2` tries `p2`
    - if `p1` fails then `try p1` fails and does not consume input (backtrack to original position in input stream)
    - `try p1 <|> p2` tries `p2` whenever `p1` fails

Details on Backtracking during Parsing

Simple ARI Parser (Demo08_Parser_ARI_Do_Blocks)

```
lexeme p = do
  a <- p
  spaces
  return a

identChar = noneOf " \t\n();:"
identifier = lexeme $ many1 identChar

term = variable <|> funapp

variable = do
  i <- identifier
  return $ Var i

charS c = do
  _ <- lexeme (char c)
  return ()
```

```
funapp = do
  charS '('
  f <- identifier
  ts <- many term
  charS ')'
  return $ Fun f ts

rule = do
   try $ do
      charS '('
      exactlyS "rule"
   l <- term
   r <- term
   charS ')'
   return (l,r)

exactlyS s = lexeme $ try $ do
  _ <- string s
  notFollowedBy identChar <?> "..." ++ s
```

Explanations

- `lexeme`
  - `lexeme p` has the same behavior as `p`, except that trailing white space is removed
  - invariant: all parsers remove trailing white space
  - advantage: later parsers can always assume that there is no leading white space
  - only exception: the main parser has to once remove leading white space
- `charS` is just a version of `char` that strips trailing white space and does not return the resulting character
- in the `rule` parser, `try` is used to backtrack to the beginning, if the initial part is not of shape `(␣*rule` where `rule` cannot be extended into a longer identifier
- to ensure the latter we use `exactlyS`, which basically is using `string "rule"` followed by the combinator `notFollowedBy`; this combinator usage enforces that no identifier character is present after `"rule"`
  - `"rule a"` is accepted by `exactlyS "rule"`, and one jumps to the beginning of `"a"`
  - `"rules a"` is not accepted by `exactlyS "rule"`, and one jumps back to the beginning of the text, complaining about `"...rule"`

When to Use Try

- with `try` and `<|>` one can easily write inefficient parsers
- `try` gives rise to backtracking, and this can become expensive
- example: detect cases of $(ab)^* \cup \{a,b\}^* c$

```
pQuadratic = try (string "ab" >> pQuadratic)
   <|> (eof >> return "(ab)^*")
   <|> (many (oneOf "ab") >> string "c" >> return "end in c")
```

- solution: close try-blocks, as soon as the applicable rule has been determined
- equivalent parser with linear runtime where only the first `try` has been changed

```
pLinear = (try (string "ab") >> pLinear)
   <|> (eof >> return "(ab)^*")
   <|> (many (oneOf "ab") >> string "c" >> return "end in c")
```

- reason for linear time is the behavior of `<|>`
  - whenever `p1` in `p1 <|> p2` consumes at least one character, then `p2` is not tried
  - consequently, if input starts with `"ab"`, then other alternatives are not tried in `pLinear`

Example: Small Try-Blocks in ARI Parser

- have a look at the rule parser again

```
rule = do
   try $ do
      charS '('
      exactlyS "rule"
   l <- term
   r <- term
   charS ')'
   return (l,r)
```

- `try` is closed after keyword `"rule"` has been detected
  - hence, after reading `(rule␣`the applied parser is fixed
- one can define similar parsers `p1,...,pn`, for each function symbol in a TRS
  - hence, `choice [p1,...,pn]` will quickly select the correct parser for `(fNameI t1 ... tk)`, namely after reading `(fNameI␣`, without major backtracking

Applicative Functors

Applicative Functors, Applicative Style

- have a look at an excerpt of a previous parser
  `parse1 = many (oneOf "ab") >> string "c" >> return "end in c"`
- here, >> is used do write the parser more succinctly
- alternative without >>

```
parse2 = do
  _ <- many (oneOf "ab")
  _ <- string "c"
  return "end in c"
```

- observation: we often invoke several parsers, but only some of them contribute to the parsed result
- >> is only one possible way to combine results: throw away result of left parser
- aim: more flexible combinations
- solution: use
  - applicative functors
  - applicative style

Applicative Functors, Difference to Functors

- known: monads have more structure than functors
- applicative functors are between monads and functors

```
class Functor f => Applicative f where
  (<*>) :: f (a -> b) -> f a -> f b
  pure :: a -> f a
```

- applicative functors are stronger than `Functors`: it is possible to lift n-ary functions to a sequence of n elements of an applicative functor, which is not possible with ordinary functors
  - $n = 2$
    `liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c`
    `liftA2 g x y = (pure g <*> x) <*> y`
    - note the partial application: `pure g <*> x :: f (b -> c)`
    - since <*> associates to the left, one just writes `pure g <*> x <*> y`
  - arbitrary n: `pure g <*> x1 <*> x2 <*> ... <*> xn`

Applicative Functors: Laws

- laws
  - `pure id <*> v = v` (identity)
  - `pure g <*> pure x = pure (g x)` (homomorphism)
  - `pure (.) <*> u <*> v <*> w = u <*> (v <*> w)` (composition)
  - `u <*> pure y = pure ($ y) <*> u` (interchange)
- consequence: `fmap g x = pure g <*> x`
  so `fmap` can be implemented via `pure` and <*>
- note the similarity and difference of type of `fmap`, <$> and <*>

```
(<$>), fmap ::   (a -> b) -> f a -> f b
(<*>)       :: f (a -> b) -> f a -> f b
```

- as we have seen, this small change is sufficient to allow arbitrary liftings of n-ary functions into the applicative functor

Towards Programming in Applicative Style

- we have already seen that sequences of <*> can combine results
- sometimes it is helpful to disregard some of the results, while still having the effect of the functor
- therefore, there are several combinators, all with fixity declaration `infixl 4`
- in general, operators with a one sided arrow symbol > use only the result from that side
- all types with * assume `Applicative f`, all with $ assume `Functor f`

```
(<*>) :: f (a -> b) -> f a -> f b
(<*)  ::        f a -> f b -> f a
(*>)  ::        f a -> f b -> f b
(<$>) ::   (a -> b) -> f a -> f b
(<$)  ::          a -> f b -> f a
```

- example implementations

```
(<$) = fmap . const
u (*>) v = (id <$ u) <*> v
```

Programming in Applicative Style

- combine the combinators of previous slide for more succinct code
- once one gets familiar with these, this does not hinder readability
- example: live demo to switch from Demo08_Parser_ARI_Do_Blocks to Demo08_Parser_ARI_Applicative
- example explanation of function application parser:

```
funapp = Fun <$> (charS '(' *> identifier) <*> many term <* charS ')'
```

  - `charS '(' *> identifier` consumes (fName
    - since we are not interested in the open parenthesis, the result of the left parser is ignored by *>
    - result of parser will be just fName
  - `Fun <$> (charS '(' *> identifier)`
    - parsing is identical, but result will now be Fun fName, a partially applied constructor
  - `Fun <$> (charS '(' *> identifier) <*> many term`
    - additionally, many terms ts are parsed and result will be the term Fun fName ts
  - `Fun <$> (charS '(' *> identifier) <*> many term <* charS ')'`
    - a closing parenthesis is parsed, but this has no impact on the result, since <* looks to the left

Applicative Functors and Monads

- every Monad is an applicative functor

```
class Functor f => Applicative f where
  pure  :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
  (*>)  :: f a ->        f b -> f b
class Applicative m => Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
  return = pure
```

- monads are stronger than applicative functors
  - `(*>) = (>>),  a1 <*> a2 = a1 >>= (\ f -> a2 >>= (\ x -> return (f x)))`
  - consider a computation involving n (monadic or functor) values
    - for applicative functors, the computation of `f <$> v1 <*> v2 <*> ... <*> vn` is possible, but each vi is computed independently, i.e., vi may not look into the results of v1,...,vi-1
    - this is in contrast to monads, where this is possible:

```
do { x1 <- v1; x2 <- v2 x1; ... xn <- vn x1 ... (xn - 1);
     return $ f x1 ... xn }
```

- example where monads are required: parser for terms can depend on parsed signature

Applicative Functors and Monads (Continued)

- sometimes monad laws are too restrictive, if one just wants to have an applicative functor
- example: collect errors during computations
- monad laws enforce the following implementation of >>=, so that an error in the second argument of >> is ignored, if first argument results in error

```
instance Monad (Either e) where
  return = Right
  Left e  >>= _ = Left e                -- Left e1 >> _ = Left e1
  Right x >>= f = f x
```

- just requiring an applicative functor permits an implementation that collects errors

```
instance Monoid e => Applicative (Either e) where
  pure = Right
  Left e1 <*> Left e2 = Left (e1 <> e2)  -- Left e1 *> Left e2
  Right f <*> Right x = Right $ f x      -- = Left $ e1 <> e2
  Left e1 <*> _       = Left e1
  _       <*> Left e2 = Left e2
```


Monad Transformer

Using Several Monads at Once: Monad Transformer

- sometimes, we would like to have the capabilities of several monads at once
- examples
  - use several states; solution: combine all states into one record datatype
  - use writer and state; solution: use RWS monad
  - use state and error; solution: write dedicated monad (PGM parser monad)
- last example is tedious
- better solution: use monad transformer
  - monad transformer takes a monad as input, and then adds another effect
  - example: take `Maybe` as input monad, and then add capabilities of `State` on top of it
  - monad transformers are all indicated by suffix `T`
  - `newtype StateT s m a = ...`
    this is the monad transformer to add `State` features
  - `type State s = StateT s Identity`
    the `State` monad is just the `StateT` monad transformer where one plugs in the `Identity` monad
  - `newtype Identity a = Identity { runIdentity :: a }` is the trivial monad
- most monads that have been presented are part of MTL, the monad transformer library

Review Definition of Known Monads Again

- most monads are actually defined via their corresponding monad transformers
- `data ParsecT s u m a = ...`
  `type Parsec s u = ParsecT s u Identity`
- `newtype RWST r w s m a = ...`
  `type RWS r w s = RWST r w s Identity`
- `newtype StateT s m a = ...`
  `type State s = StateT s Identity`
- ...; notable exception: for `IO` there is no `IOT` monad transformer
- with monad transformers we can easily combine multiple effects
  - `RWST r w s Maybe` combines `RWS` with `Maybe` error monad
  - `RWST r w s IO` combines `RWS` with `IO` monad
  - `StateT st (ParsecT s u m)` is monad transformer that adds `State` and `Parsec` features
  - `Identity` can always be used to terminate a stack of monad transformers, e.g.,
    `MT1 s (MT2 r (... MTn u Identity))`
- because of mentioned restriction, `IO` must always be at the inside

Example: Just using IO Monad

- write function to list all subdirectories with number of entries per directory

```
listDirectory :: FilePath -> IO [String]
listDirectory d = filter notDots <$> getDirectoryContents d
    where notDots p = not $ p `elem` [".", ".."]

countEntries1 :: FilePath -> IO [(FilePath, Int)]
countEntries1 path = do
  contents <- listDirectory path
  rest <- flip mapM contents $ \name -> do
      let newName = path </> name
      isDir <- doesDirectoryExist newName
      if isDir
        then countEntries1 newName
        else return []
  return $ (path, length contents) : concat rest
```


Example: Collect Output in Writer Monad via `WriterT`

```
countEntries2Main :: FilePath -> WriterT [(FilePath, Int)] IO ()
countEntries2Main path = do
  contents <- liftIO . listDirectory $ path
  tell [(path, length contents)]
  flip mapM_ contents $ \name -> do
      let newName = path </> name
      isDir <- liftIO . doesDirectoryExist $ newName
      when isDir $ countEntries2Main newName

countEntries2 :: FilePath -> IO [(FilePath, Int)]
countEntries2 = fmap snd . runWriterT . countEntries2Main
```

Explanations

- `countEntriesMain :: ... -> WriterT [(FilePath, Int)] IO ()`
  - since the outer type is `WriterT`, the result type is an instance of `MonadWriter [...]`
  - therefore, `tell :: [...] -> WriterT [...] m ()` is available
- `liftIO :: MonadIO m => IO a -> m a` lifts IO-actions to a corresponding monad
  - `IO` is a trivial instance of `MonadIO` where `liftIO = id`
  - there also is an instance `(Monoid w, MonadIO m) => MonadIO (WriterT w m)`; this tells us that being an `MonadIO` instance is preserved by `WriterT w`
- `when :: Applicative f => Bool -> f () -> f ()` is if-then without else: `when p s = if p then s else pure ()`
- `runWriterT :: WriterT w m a -> m (a, w)`
  - run the `WriterT` monad transformer
  - result will be in original monad `m`
  - output of writer will be made available in second component of result
  - similar to `runWriter :: Writer w a -> (a, w)`
- overall: availability of both `MonadWriter` and `IO`; run `runWriterT` to convert `WriterT w m a` into `m (a,w)`, i.e., eliminate `WriterT`

Design of MTL

- several abstract classes, e.g., `MonadWriter`, `MonadReader`, `MonadState`, `MonadIO`,...
- several monad transformers, e.g., `WriterT`, `ReaderT`, `StateT`, ...
- $n \times n$ instance declarations
  - `(Monoid w, Monad m) => MonadWriter w (WriterT w m)` — `MonadWriter` instance
  - `(Monoid w, MonadIO m) => MonadIO (WriterT w m)` — preserve `MonadIO`
  - `(Monoid w, MonadState s m) => MonadState s (WriterT w m)` preserve `MonadState`
  - ...
  - `Monad m => MonadReader r (ReaderT r m)` — `MonadReader` instance
  - `MonadIO m => MonadIO (ReaderT r m)` — preserve `MonadIO`
  - `MonadState s m => MonadState s (ReaderT r m)` — preserve `MonadState`
  - ...
- in total
  - allows flexible stacking of monad transformers: choose those transformers that are required for application
  - quite some effort to integrate new monad transformer: full implementation requires connection to all other abstract classes

Example for Stacking Monad Transformers

- example is an extension of the directory count example
  - extension 1: user must specify maximal recursion depth
  - extension 2: compute reached maximal recursion depth
- utilized monads
  - `MonadIO` is required for directory access
    - access via `liftIO :: MonadIO m => IO a -> m a`
  - use `MonadReader` to pass configuration around; that configuration stores recursion limit
    - access via `ask :: MonadReader r m => m r`
  - use `MonadState` to store the maximally reached recursion depth
    - access via `get :: MonadState s m => m s` and `put :: MonadState s m => s -> m ()`

Stacking of Monad Transformers Example – Setup

```
data AppConfig = AppConfig {
      cfgMaxDepth :: Int
    } deriving (Show)

data AppState = AppState {
      stDeepestReached :: Int
    } deriving (Show)

type App = ReaderT AppConfig (StateT AppState IO)

runApp :: App a -> Int -> IO (a, AppState)
runApp app maxDepth =
    let config = AppConfig maxDepth
        state = AppState 0
    in runStateT (runReaderT app config) state
```

Stacking of Monad Transformers Example – App

```
countEntries3Main :: Int -> FilePath -> App [(FilePath, Int)]
countEntries3Main curDepth path = do
  contents <- liftIO . listDirectory $ path
  allowedDepth <- cfgMaxDepth <$> ask
  rest <- flip mapM contents $ \name -> do
    let newPath = path </> name
    isDir <- liftIO $ doesDirectoryExist newPath
    if isDir && curDepth < allowedDepth
      then do
        let newDepth = curDepth + 1
        st <- get
        when (stDeepestReached st < newDepth) $
          put st { stDeepestReached = newDepth }
        countEntries3Main newDepth newPath
      else return []
  return $ (path, length contents) : concat rest
```


Final Steps

- wrapper for application that removes `App` type

```
countEntries3Main :: Int -> FilePath -> App [(FilePath, Int)]
runApp :: App a -> Int -> IO (a, AppState)

countEntries3 :: Int -> FilePath -> IO ([(FilePath, Int)], Int)
countEntries3 md fp =
  second stDeepestReached <$> runApp (countEntries3Main 0 fp) md
```


Limits of MTL

- when using MTL, one often can just use all features of the transformers in the stack
- there are two major exceptions
  - a single transformer occurs multiple times, e.g., `StateT Int (StateT String IO)`
    - what should be the type of `get`? return an `Int` or a `String`? how to access the other state?
  - monads outside MTL are used, where no automatic instance forwarding is available
    - example problem

```
class Monad m => MyMonad m where
  myFun :: Int -> a -> m [a]

foo :: MyMonad m => a -> ReaderT Int m a
foo x = do
  i <- ask
  {- how to invoke "xs <- myFun i x" at this point? -}
  return $ xs !! max i 5
```

- both problems can be solved by using
  `lift :: (MonadTrans t, Monad m) => m a -> t m a`
  - using `lift`, we get access to monad operations that are one level deeper in the stack
  - most (or even all) monad transformers in MTL instantiate `MonadTrans`

`MonadTrans` and `lift :: (MonadTrans t, Monad m) => m a -> t m a`

- second problem solved

```
class Monad m => MyMonad m where myFun :: Int -> a -> m [a]
foo :: MyMonad m => a -> ReaderT Int m a
foo x = do
  i <- ask
  xs <- lift $ myFun i x
  return $ xs !! max i 5
```

  - here, `myFun i x :: m [a]`, so `lift $ myFun i x :: ReaderT Int m [a]`
- first problem solved

```
bar :: StateT Int (StateT String IO) ()
bar = do
  (x :: Int) <- read <$> liftIO getLine
  put x                          -- outer StateT
  (s :: String) <- lift $ get    -- inner StateT
  liftIO $ putStrLn s
```

Design Decision

- in second problem from previous slide, one has two alternatives
- solution via `lift`
  - advantage: no instance declarations are required
  - disadvantage: application code needs to insert `lift`
- solution by writing instance declarations
  - disadvantage: a lot of boilerplate code has to be written ($n \times n$ problem)
  - advantage: more comfort for the user – fewer manual liftings
- preferable solutions depends on number of required liftings

Exercises

- Convince yourself that the order of monad transformers matters. Use two different monad transformer stacks to run the following code, so that the result is different. Provide the wrapper functions and explain the difference.

```
testApp :: (MonadError String m, MonadWriter [Bool] m) => m Int
testApp = do
  tell [True]
  throwError "bar"
  return 5
```

- Use monad transformers to design an SMT encoding of the lexicographic path order; details: see `Exercise08*.hs`

Literature

- Real World Haskell, Chapters 16 and 18