universität innsbruck

Automata and Logic | WS 2024/2025 | LVA 703026 + 703027

Week 2+3 | October 25, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on October 25.

Exercises

⟨2⟩ 1. Consider the following NFA$_\epsilon$ N:

		ϵ	a	b	c
$\to$	1	$\varnothing$	$\{1\}$	$\{2\}$	$\{1,3\}$
	2	$\{1\}$	$\{2\}$	$\{3\}$	$\varnothing$
	$3F$	$\{2,3\}$	$\{3\}$	$\varnothing$	$\{1\}$

   (a) Compute the ϵ-closure of each state.

   (b) List all strings in $L(N)$ of length two or less.

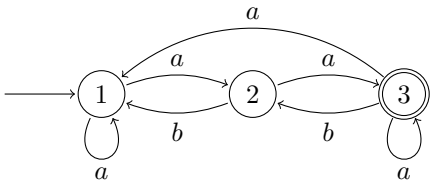
   (c) Transform N into an equivalent DFA without inaccessible states.

⟨2⟩ 2. Give regular expressions for each of the following subsets of $\{a,b\}^*$:

   (a) $\{x \mid \#a(x) = 1\}$

   (b) $\{x \mid x \text{ does not contain the substring } ab\}$

⟨2⟩ 3. Transform the following automaton into an equivalent regular expression, using the method on slide 10 of lecture 3:



⟨2⟩ 4. (a) Prove that regular sets are closed under string reversal.

   (b) Prove that every regular set is accepted by an NFA with exactly one final state.

⟨2⟩ 5. Provide concrete algorithms for the decision problems on slide 24 of lecture 3. Given an estimate of the complexity of your algorithms in terms of the size (number of states, length of input string) of the instance.

Bonus Exercise

⟨5⟩ 6. Given a set A of strings, define the set $\mathrm{LT}(A) = \{y \mid xy \in A \text{ and } |x| = 2|y| \text{ for some string } x\}$.

   (a) Prove that $\mathrm{LT}(A)$ is regular whenever A is regular.

   (b) Does the converse hold?