universität innsbruck

Automata and Logic	WS 2024/2025	LVA 703026 + 703027
Week 4+5		November 8, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on November 8.

Exercises

⟨2⟩ 1. Consider the following DFA M:

		a	b
→	$1F$	3	5
	$2F$	8	7
	3	7	2
	4	6	2
	5	1	8
	6	2	3
	7	1	4
	8	5	1

(a) Determine which states are accessible.

(b) Compute the equivalence classes of the indistinguishability relation $\approx$.

(c) Compute the minimum-state DFA for $L(M)$.

⟨2⟩ 2. Give WMSO formulas that define the following regular sets over $\Sigma = \{a, b, c\}$.

(a) $\{abxc \mid x \in \Sigma^*\}$

(b) $\Sigma^* - L(a^*b^*c^*)$

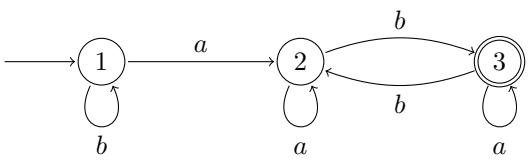
⟨3⟩ 3. Which of the following sets are regular and which are not? Provide justification.

(a) $A = \{0^m 1^n \mid m \neq n\}$

(b) $B = \{0^m 1^{m+n} \mid m \leqslant n\}$

(c) $C = \{0^m 1^n 2^k \mid m \neq n \text{ or } m \neq k\}$

⟨1⟩ 4. Consider the following DFA M:



Construct a WMSO formula φ such that $L(\varphi) = L(M)$.

⟨2⟩ 5. Consider the regular expression $\alpha = a(a + b)^*b$.

(a) Compute the minimum-state DFA M for $L(\alpha)$.

(b) Give a regular expression for each of the equivalence classes of the Myhill–Nerode relation $\equiv_M$ of M.

Bonus Exercise

⟨5⟩ 6. (a) Let N be an NFA with n states such that $L(N) \neq \varnothing$. Prove that $L(n)$ contains a string x with $|x| < n$.

(b) Construct a NFA N over $\Sigma = \{a\}$ with n states such that $|x| > n$ for the shortest string $x \notin L(N)$.

(c) Give a construction for arbitrarily large NFAs showing that the length of the shortest rejected string can be exponential in the number of states.