universität innsbruck

Automata and Logic	WS 2024/2025	LVA 703026 + 703027
Week 7		November 22, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on November 22.

Exercises

⟨2⟩ 1. (a) Express $x \equiv y \pmod{k}$ for any $k > 0$ as Presburger arithmetic formula.

(b) Use MONA to get a minimal solution for $x \equiv y \pmod{3} \wedge y < x$. Note that you can use `pconst(k)` to get the set representation of the constant natural number k. Furthermore, it is convenient to use the `pred` keyword to define predicates.

⟨2⟩ 2. Construct an automaton A_φ such that $L(A_\varphi) = L(\varphi)$ for the Presburger formula $\varphi = \exists y.x = 3y + 1$. Check that your automaton indeed accepts the representations of 1, 4, 7 and rejects representations of 2 and 3.

⟨3⟩ 3. Adapt the construction on slide 21 such that A_φ accepts representations of solutions for a given *inequality* $a_1x_1 + \cdots + a_nx_n \leqslant b$. Illustrate your algorithm on the inequality $3x - 2y \leqslant 1$.[1]

⟨3⟩ 4. Prove the second part of the theorem on slide 21. I.e., show the following: A string x is accepted by the automaton A_φ if and only if $\underline{x}$ is a solution for the equation $a_1x_1 + \cdots + a_nx_n = b$.

[1] Solutions obtained by implementation are welcome! In that case please submit code with sufficient documentation to execute and understand its functionality.