Automata and Logic | WS 2024/2025 | LVA 703026 + 703027

Week 8 | November 29, 2024

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on November 29.

Exercises

⟨2⟩ 1. Let $U_1, U_2, U_3 \subseteq \Sigma^*$. Determine whether the following equalities hold. Explain your answers.

   (a) $(U_1 \cup U_2) \cdot U_3^\omega = (U_1 \cdot U_3^\omega) \cup (U_2 \cdot U_3^\omega)$

   (b) $U_1 \cdot (U_2 \cup U_3)^\omega = (U_1 \cdot U_2^\omega) \cup (U_1 \cdot U_3^\omega)$

   (c) $(U_1^* \cup U_2^*)^\omega = (U_1 \cup U_2)^\omega$

   (d) $(U_1^* \cdot U_2^*)^\omega = (U_1 \cdot U_2)^\omega$

⟨3⟩ 2. Give Büchi automata accepting the following ω-regular sets. Which of these are accepted by a DBA?

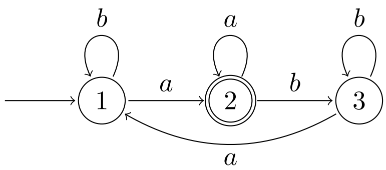
   (a) $\{ab, ba\}^\omega$

   (b) $\{x \in \{a,b,c\}^\omega \mid x \text{ does not contain the substring } ab\}$

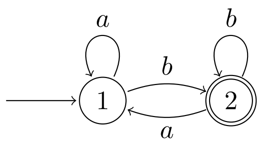
   (c) $\{x \in \{a,b,c\}^\omega \mid \text{there are at least two } b\text{'s between each two successive } a\text{'s in } x\}$

⟨2⟩ 3. Consider the following NBAs over $\Sigma = \{a, b\}$:

M_1:



M_2:



   (a) For each of the following sets give an infinite string that is contained in it:

   $L(M_1) - L(M_2)$ $\qquad$ $L(M_2) - L(M_1)$ $\qquad$ $L(M_1) \cap L(M_2)$

   (b) Apply the product construction from slide 26 to obtain an NBA M such that $L(M) = L(M_1) \cap L(M_2)$.

⟨2⟩ 4. Prove or disprove the following statement: For every Büchi automaton M there exists a Büchi automaton M' such that $L(M) = L(M')$ and M' has a single accepting state.

⟨1⟩ 5. Show that it is decidable whether $L(M) = \varnothing$ for a given Büchi automaton M.

Bonus Exercise

⟨5⟩ 6. Show that there is a singleton set $A \subseteq \Sigma^\omega$ which is not ω-regular.