universität innsbruck

WS 2024/2025

Functional Programming

Week 2 – Tree Shaped Data and Datatypes

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Last Lecture

- algorithm (can be informal) vs. program (concrete programming language)
- Haskell script (code, program, ...), e.g., `program.hs`
  ```
  fahrenheitToCelsius f = (f - 32) * 5 / 9
  ```
  consists of function definitions that describe input-output behaviour
- function- and parameter-names have to start with lowercase letters
- read-eval-print loop (REPL):
  load script, enter expressions and let these be evaluated

```
$ ghci program.hs
... welcome message ...
Main> fahrenheitToCelsius (3 + 20) - 7
-12.0
Main> ... further expressions ...
...
Main> :q
```


Structured Data

Different Representations of Data

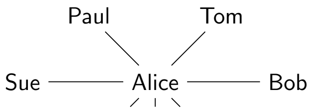
- some (abstract) element can be represented in various ways
- example: numbers
  - roman: XI
  - decimal: 11
  - binary: 1011
  - English: eleven
  - tally list: |||||||||||
- fact: algorithms depend on concrete representation
- example: addition
  - decimal + binary: process digits of both numbers from right to left

    ```
      7823
    +  909
    ------
      8732
    ```
  - tally list: just write the two numbers side-by-side (||| + || = |||||)
  - roman: algorithm? (IV + IX = XIII)
  - English: not well-suited (twentynine + two = thirtyone)
- in Haskell: numbers are built-in, representation not revealed to user

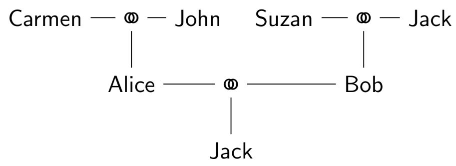
Different Representations of Data – Continued

- representation must be chosen appropriately
- example: person
  - photographer:

  - social analysis:

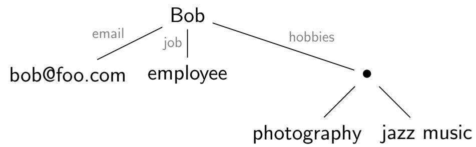


  - advertizing: Bob (bob@foo.com, employee, hobbies: photography, jazz music, . . .)
  - genealogist:

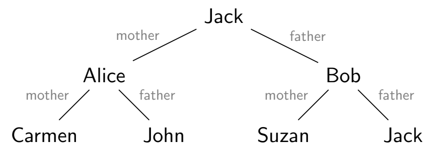


Tree Shaped Data

- in functional programming most of the data is tree shaped
- a tree
  - has exactly one root node
  - can have several subtrees; nodes without subtrees are leaves
  - nodes and edges can be labeled
- in computer science, trees are usually displayed upside-down
- examples from previous slide
  - advertizing:

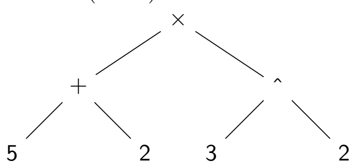


  - genealogist:



Expressions = Trees

- mathematical expressions can be represented as trees
- example
  - expression in textual form: $(5 + 2) \times 3\hat{\ }2$
  - expression as tree



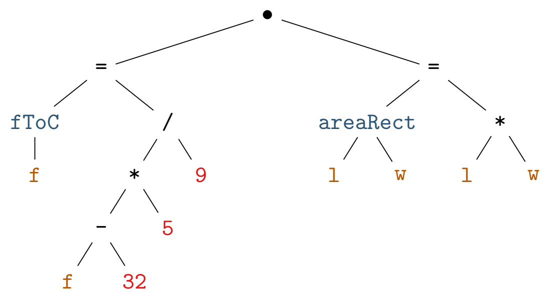
- remarks
  - the process of converting text into tree form is called parsing
  - operator precedences (^ binds stronger than ×, and × binds stronger than +) and parentheses are only required for parsing
    - parsing $(5 + 2) \times (3\hat{\ }2)$ results in tree above
    - $5 + 2 \times 3\hat{\ }2$ and $((5 + 2) \times 3)\hat{\ }2$ represent other trees
  - algorithm of calculator
    - convert textual input into tree
    - evaluate the tree bottom-up, i.e., start at leaves and end at root

Programs = Trees

- programs can be represented as trees, too: abstract syntax tree
- example
  - program in textual form

```
-- some comment
fToC f = (f - 32) * 5 / 9
areaRect l w = l * w
```

  - abstract syntax tree (draft)



  - comments and parentheses are no longer present in syntax tree

Tree Shaped Data

- many programs deal with tree shaped data
- examples
  - calculator evaluates expression tree
  - compiler translates abstract syntax tree into machine code
  - search engine translates query into HTML (tree shaped)
  - contact application manages tree shaped personal data
  - file systems are organised as trees
- trees as mental model or representation of data is often suitable
- good news: processing tree shaped data is well-supported in functional programming
- next lecture: define functions on trees
- this lecture: restriction of trees via types

Types

Types

- functions are often annotated by their domain and codomain, e.g.,
  - $(!) : \mathbb{N} \to \mathbb{N}$
  - $(/) : \mathbb{R} \times (\mathbb{R} \setminus \{0\}) \to \mathbb{R}$
  - $log_2 : \mathbb{R}_{>0} \to \mathbb{R}$
- domain and codomain provide useful information
  - domain: what are allowed inputs to a function
  - codomain: what are potential outputs of the function
- aim: specify domains and codomains of (Haskell-)functions
- notions
  - elements or values
    - maths: 5, 8, π, $-\frac{3}{4}$, ...
    - Haskell: `5`, `8`, `3.141592653589793`, `-0.75`, ..., `"hello"`, `'c'`, ...
  - sets of elements to specify domain or codomain, in Haskell: types
    - maths: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{Q} \setminus \{0\}$, ...
    - Haskell: `Integer` ($\mathbb{Z}$), `Double` ($\mathbb{R}$), `String`, `Char`, ...

Typing Judgements

- in maths, we write statements like $7 \in \mathbb{Z}$, $7 \in \mathbb{R}$, $0.75 \notin \mathbb{Z}$
- similarly in Haskell, we can express that a value or expression has a certain type via typing judgements
  - format: `expression :: type`
  - examples
    - `7 :: Integer` or `7 :: Double`
    - `'c' :: Char`
- that an expression indeed has the specified type is checked by the Haskell compiler
  - if an expression has not the given type, a type error is displayed
  - examples which raise an error
    - `7 :: String` or `0.75 :: Integer` or `'c' :: String`
    - `(7 :: Integer) :: Double`
  - remarks
    - unlike in maths where $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$, in Haskell the types `Integer` and `Double` are not subtypes of each other
    - although some expressions can have both types (e.g., `7 + 5`), in general numbers of different types have to be converted explicitly
    - once a typing judgement is applied, the type of that expressions is fixed

Typing of Haskell Expressions

- not only values but also functions have a type, e.g.,
  - `(/) :: Double -> Double -> Double`
  - `(+) :: Integer -> Integer -> Integer`
  - `(+) :: Double -> Double -> Double`
  - `head :: String -> Char`

  remarks
  - a function can have multiple types, e.g., `(+)`
  - limited expressivity, e.g. `(/) :: Double -> Double \ {0} -> Double` not allowed
- type checking enforces that in all function applications, type of arguments matches input-types of function
- example: consider expression `expr1 / expr2`
  - recall: `(/) :: Double -> Double -> Double`
  - it will be checked that both `expr1` and `expr2` have type `Double`
  - type of the overall expression `expr1 / expr2` will then be `Double`
- examples
  - `5 + 3 / 2` ✔
  - `5 + '3'` or `5.2 + 0.8 :: Integer` ✘

Static Typing

- Haskell performs static typing
- static typing: types will be checked before evaluation
  (by contrast, dynamic typing checks types during evaluation)
- when loading Haskell script
  - check types of all function definitions `someFun x ... z = expr`:
    check that lhs `someFun x ... z` has same type as rhs `expr`
  - consequence: expressions cannot change their type during evaluation
- when entering expression in REPL: type check expression before evaluation
- benefits
  - no type checking required during evaluation
  - no type errors during evaluation

Built-In Types – A First Overview

- numbers
  - `Integer` – arbitrary-precision integers
  - `Int` – fixed-precision integers with range at least $\{-2^{28}, \ldots, 2^{28} - 1\}$ (`-100`, `0`, `999`)
  - `Float` – single-precision floating-point numbers (`-12.34`, `5.78e36`)
  - `Double` – double-precision floating-point numbers
- characters and text
  - `Char` – a single character (`'a'`, `'Z'`, `' '`)
  - `String` – text of arbitrary length (`""`, `"a"`, `"The answer is 42."`)
  - some characters have to be escaped via the backslash-symbol \:
    - `'\t'` and `'\n'` – tabulator and new-line
    - `'\"'` and `'\''` – double- and single quote
    - `'\\'` – the backslash character
    - example: in the program
      `text = "Please say \"hello\"\nwhenever you enter the room"`
      the string `text` corresponds to the following two lines:
      ```
      Please say "hello"
      whenever you enter the room
      ```
- `Bool` – yes/no-decisions or truth-values (`True`, `False`)

Datatypes

Current State

- each value and function in Haskell has a type
- types are used to define input and output of function
- example: `fahrenheitToCelsius :: Double -> Double`
- built-in types for numbers, strings, and truth values
- missing: how to define types that describe tree shaped data?
- solution: definition of (algebraic) datatypes

Datatype Definitions

- recall: a tree consists of a (labelled) root and 0 or more subtrees
- a datatype definition defines a set of trees by specifying all possible labelled roots together with a list of allowed subtrees
- Haskell scripts can contain many datatype definitions of the form

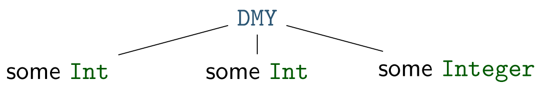
```
data TName =
    CName1 type1_1 ... type1_N1
  | ...
  | CNameM typeM_1 ... typeM_NM
  deriving Show
```

where
  - `data` is a Haskell keyword to define a new datatype
  - `TName` is the name of the new type; type-names always start with capital letters
  - `CName1`,...,`CNameM` are the labels of the permitted roots; these are called constructors and have to start with capital letters
  - `typeI_J` can be any Haskell type, including `TName` itself
  - | is used as separator between different constructors
  - `deriving Show` is required for displaying values of type `TName`

Example Datatype Definition – Date

```
data Date = -- name of type
  DMY       -- name of constructor
    Int     -- day
    Int     -- month
    Integer -- year
  deriving Show
```

- here, there is only one constructor: `DMY`
- for day and month the precision of `Int` is sufficient
- the values of the type `Date` are exactly trees of the form



- in Haskell, these trees are built via the constructor `DMY`; `DMY` is a function of type `Int -> Int -> Integer -> Date` that is not evaluated
- example value of type `Date`: `DMY 14 10 2024`

Example Datatype Definition – Person

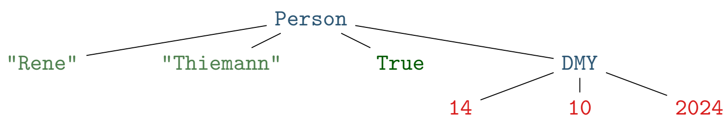
```
data Person = -- name of type
  Person      -- constructor name can be same as type name
    String    -- first name
    String    -- last name
    Bool      -- married
    Date      -- birthday
  deriving Show
```

- reuse of previously defined types is permitted, in particular `Date`
- this leads to trees with more than one level of subtrees
- example program that defines a person (and an auxiliary date)

```
today = DMY 14 10 2024
myself = Person "Rene" "Thiemann" True today
-- is the same as
myself = Person "Rene" "Thiemann" True (DMY 14 10 2024)
```

Trees and Their Textual Representation

- in Haskell, trees have to be entered in a textual form, and trees are also output in textual form
- to define a tree with root constructor `C` and subtrees `t1, ..., tN`
  - one writes `C (t1) ... (tN)`;
  - if some `tI` is not a composed expression, then one can omit the parenthesis around `tI`;
  - this format is the same as for function applications
- example



- `Person "Rene" "Thiemann" True (DMY 14 10 2024)` ✔
- `Person "Rene" "Thiemann" (5 > 3) (DMY 14 (length "0123456789") 2024)` ✔
- `Person ("Rene", "Thiemann", True, DMY (14, 10, 2024))` ✘
- `Person "Rene" "Thiemann" True DMY 14 10 2024` ✘

Example Datatype Definition – `Vehicle`

```
data Brand = Audi | BMW | Fiat | Lamborghini deriving Show
data Vehicle =
    Car Brand Double -- horsepower
  | Bicycle
  | Truck Int -- number of wheels
  deriving Show
```

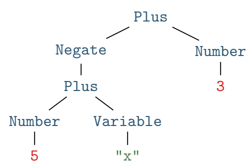
- `Brand` just defines 4 car brands; all "trees" of type `Brand` consist of a single node; such datatypes are called enumerations
- there are three kinds of `Vehicle`s, each having a different list of types
- example expressions of type `Vehicle`:

```
Car Fiat (60 + 1)
Car Audi 149.5
Bicycle
Truck (-7) -- types don't enforce all sanity checks
```


Example Datatype Definition – `Expr`

```
data Expr =
    Number Integer
  | Variable String
  | Plus Expr Expr
  | Negate Expr
  deriving Show
```

- type `Expr` models arithmetic expressions with addition and negation
- `Expr` ia a recursive datatype: `Expr` is defined via `Expr` itself
- recursive datatypes contain values (trees) of arbitrary large height
  - expression $(-(5 + x)) + 3$ in Haskell (as value of type `Expr`):
    `Plus (Negate (Plus (Number 5) (Variable "x"))) (Number 3)`
  - expression as tree

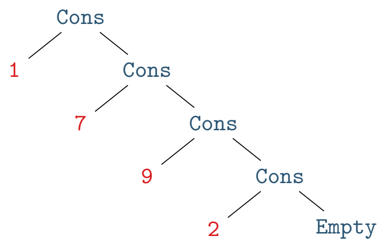


Example Datatype Definition – Lists

- lists are just a special kind of trees, e.g., lists of integers

```
data List =
    Empty
  | Cons Integer List
  deriving Show
```

- example representation of list $[1, 7, 9, 2]$
  - in Haskell: `Cons 1 (Cons 7 (Cons 9 (Cons 2 Empty)))`
  - as tree:



Summary

- mental model: data = tree shaped data
- type = set of values; restricts shape of trees
- built-in types for numbers and strings
- user-definable datatypes, e.g., for expressions, lists, persons

```
data TName =
    CName1 type1_1 ... type1_N1
  | ...
  | CNameM typeM_1 ... typeM_NM
  deriving Show
```

- next lecture: function definitions on trees