



Advanced Functional Programming

Week 2 – Type-Checking and Type-Inference

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Type-Checking and Type-Inference

Static and Dynamic Type-Checking

- every Haskell expression is type-checked
- static type-checking: ill-typed expressions are detected at compile time
- big advantage: well-typed programs cannot go wrong (w.r.t. typing errors)
 - evaluation cannot change the type of an expression
 - example: if $f :: \text{String} \rightarrow \text{Int}$ and $e :: \text{String}$, then $f\ e :: \text{Int}$, independent of evaluation
 - conclusion: detect type-errors at compile-time, erase types at runtime
- alternative: dynamic type-checking (e.g., Python)
 - dynamic: types are determined at run-time
 - consider $f\ x = \text{if } x > 3.1415 \text{ then "foo" else } 5$
 - now evaluate $f\ (\text{approxPi } 1000) - 2$
 - only after evaluation of $\text{approxPi } 1000$ we can determine the Boolean $\text{approxPi } 1000 > 3.1415$
 - this Boolean decides whether $f\ (\text{approxPi } 1000)$ evaluates to the string "foo" or to the number 5
 - only then we know whether we will get a type error ("foo" - 2) or no type-error (5 - 2)

```
def f(x): return ("foo" if x > 3.1415 else 5)

# pi = 4 * (1 - 1/3 + 1/5 - 1/7 + 1/9 - ...)
def approxPi(x):
    p = 1
    y = 3
    m = -1
    while (x > 0):
        x -= 1
        p += m/y
        y += 2
        m *= -1
    return (4 * p)
```

```
# question: do the following python functions lead to type-errors?
def test1(): return f(approxPi(1000)) - 2
def test2(): return f(approxPi(1001)) - 2
```

Static Type-Checking and Type-Inference

- **type-checking**: given expression e , context Γ and type ty , determine whether

$$\Gamma \vdash e :: ty \quad (\text{e has type } ty \text{ in context } \Gamma)$$

using some typing rules, e.g., the ones of Haskell, ML, ...

- **context** Γ : stores types of previously defined variables, functions and constructors
 - Γ might contain $(:) :: a \rightarrow [a] \rightarrow [a]$, $\text{True} :: \text{Bool}$, $\text{id} :: a \rightarrow a$, ...
 - we often just write $e :: ty$ instead of $\Gamma \vdash e :: ty$ if choice of Γ is clear
- **type-inference**: given expression e and context Γ , determine a **most general type** (aka **principal type**) of e or report non-typability
 - **most general**: $ty1$ is more general than $ty2$ if there is some type-substitution τ such that $ty1\tau = ty2$
 - a is more general than any type ty , choose $\tau := \{a/ty\}$
 - $a \rightarrow \text{Int} \rightarrow b$ is more general than $[b] \rightarrow \text{Int} \rightarrow \text{String}$, take $\tau := \{a/[b], b/\text{String}\}$
 - $a \rightarrow \text{Int} \rightarrow b$ is not more general than $\text{Char} \rightarrow [\text{Int}] \rightarrow \text{Char}$
- Haskell performs type-inference where type-inference will be applied twice
 - on function definitions in Haskell programs
 - on each expression before it is evaluated in `ghci`

Non-Deterministic Type-Checking Algorithm

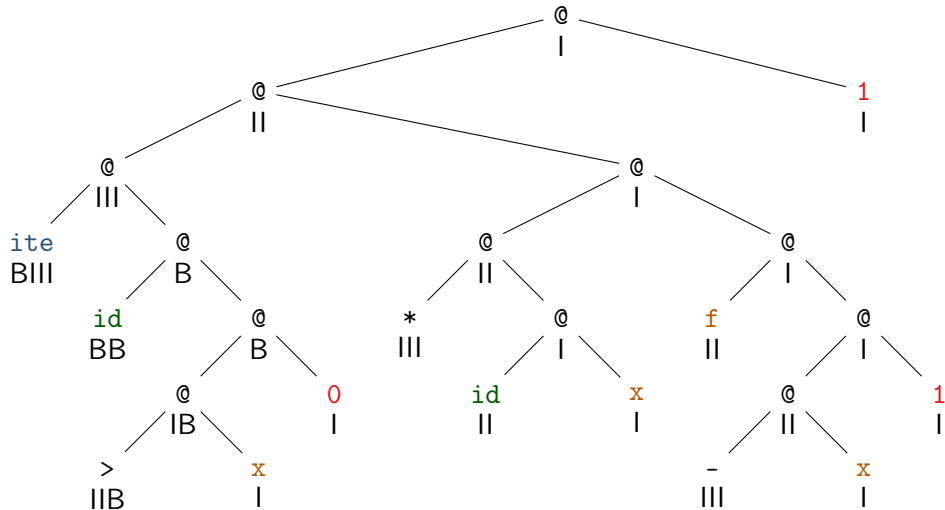
- note: we restrict to expressions built from variables, constants and applications
- algorithm to type-check new definition of $f \ p1 \ \dots \ pn = rhs$ in context Γ
 - guess a type for f of shape $ty1 \rightarrow \dots \rightarrow tyn \rightarrow ty$
(or take a user-defined type annotation for f)
 - guess a type for each variable x in the defining equation
 - for each constant $c \neq f$ that appears in the defining equation, guess an instance w.r.t. Γ
 - e.g., if $id :: a \rightarrow a \in \Gamma$, then each occurrence of id can choose a different substitution, e.g., $id :: Int \rightarrow Int$ and $id :: Bool \rightarrow Bool$
 - define a local context Γ' that extends Γ by all guesses
 - type-check definition of f by checking $\Gamma' \vdash f \ p1 \ \dots \ pn :: ty$ and $\Gamma' \vdash rhs :: ty$ by recursion on the expressions
 - $\Gamma' \vdash xf :: t$ if $xf :: t \in \Gamma'$ if xf is a variable or $xf = f$
 - $\Gamma' \vdash c :: t$ if $c :: t \in \Gamma'$ according to guessed instance for each constant $c \neq f$
 - **type-check all applications**
$$\frac{\Gamma' \vdash e1 :: t1 \rightarrow t2 \quad \Gamma' \vdash e2 :: t1}{\Gamma' \vdash e1 \ e2 :: t2}$$
- finally, store $f :: ty1 \rightarrow \dots \rightarrow tyn \rightarrow ty$ in Γ

Example

- `f x = if id (x > 0) then id x * f (x - 1) else 1`
 - guess `f :: Int -> Int`
 - guess `x :: Int`
 - instantiate `if-then-else :: Bool -> Int -> Int -> Int`
(`if-then-else :: Bool -> a -> a -> a ∈ Γ`)
 - instantiate `id :: Bool -> Bool` (`id :: a -> a ∈ Γ`)
 - instantiate `(>) :: Int -> Int -> Bool`
 - instantiate `0 :: Int`
 - instantiate `id :: Int -> Int`
 - instantiate `*` `:: Int -> Int -> Int`
 - instantiate `-` `:: Int -> Int -> Int`
 - instantiate `1 :: Int`
 - instantiate `1 :: Int`
- on next slide, abbreviate `Int -> Int -> Bool` by `IIB`, etc.

Example Typing

`f x = if id (x > 0) then id x * f (x - 1) else 1`



Example Typing

```
f x = if id (x > 0) then id x * f (x - 1) else 1
```

- guesses work out
 - we assumed $f :: \text{Int} \rightarrow \text{Int}$ and $x :: \text{Int}$
 - then lhs $f\ x :: \text{Int}$ and rhs $\text{if } \dots :: \text{Int}$
 - so $f :: \text{Int} \rightarrow \text{Int}$ is added to Γ
- guesses might be too specific, but it is possible to guess most general type

Next Step – Type Inference

- avoid guesses
- compute most generic types instead
- algorithm of Hindley and Milner
 - use very generic types first (which might be too generic)
 - setup constraints
 - solve constraints and thereby specialize initial types

Example Typing – Inferring a Most General Type

```
map f [] = []
```

```
map f (x : xs) = f x : map f xs
```

- n -ary function gets type $a_0 \rightarrow \dots \rightarrow a_n$ with type-variables $a_0, \dots, a_n$
 - $\text{map} :: a_1 \rightarrow a_2 \rightarrow a_3$
- each variable in defining equation gets assigned fresh type-variable
 - $f :: a_4$ (simplication; usually, one would distinguish the two f s in both defining equations)
 - $x :: a_5$
 - $xs :: a_6$
- instantiate all type-variables in type of constants by fresh type-variables
 - instantiate $[] :: [a_7]$
 - instantiate $[] :: [a_8]$
 - instantiate $(:) :: a_9 \rightarrow [a_9] \rightarrow [a_9]$
 - instantiate $(:) :: a_{10} \rightarrow [a_{10}] \rightarrow [a_{10}]$

Example Typing – Setting Up Constraints

```
map f [] = []
```

```
map f (x : xs) = f x : map f xs
```

- setup from previous slide
 - $\text{map} :: a1 \rightarrow a2 \rightarrow a3, f :: a4, x :: a5, xs :: a6$
 - $[] :: [a7], [] :: [a8], (:) :: a9 \rightarrow [a9] \rightarrow [a9],$
 $(:) :: a10 \rightarrow [a10] \rightarrow [a10]$
- further assign type-variables to all non-atomic subexpressions of patterns and rhss
 - $(:) x :: b1, x : xs :: b2, f x :: b3, (:) f x :: b4, \text{map } f :: b5,$
 $\text{map } f xs :: b6, f x : \text{map } f xs :: b7$
- finally add constraints to ensure applicability of typing rules
 - $a1 = a4$, first argument of `map` in lhss of equations
 - $a2 = [a7], a2 = b2$, second argument of `map` in lhss of equations
 - $a3 = [a8], a3 = b7$, return type of `map` equals type of rhss in both equations
 - consider each application $e1\ e2$
 - lookup types for $e1 :: t1, e2 :: t2$, and $e1\ e2 :: t3$
 - add constraint $t1 = t2 \rightarrow t3$

Example Typing – Final Constraints

```
map f [] = []
```

```
map f (x : xs) = f x : map f xs
```

- setup

- `map :: a1 -> a2 -> a3, f :: a4, x :: a5, xs :: a6`
- `[] :: [a7], [] :: [a8], (:) :: a9 -> [a9] -> [a9],
 (:) :: a10 -> [a10] -> [a10]`
- `(:) x :: b1, x : xs :: b2, f x :: b3, (:) f x :: b4, map f :: b5,
 map f xs :: b6, f x : map f xs :: b7`

- constraints

- `a1 = a4, a2 = [a7], a2 = b2, a3 = [a8], a3 = b7`
- `a9 -> [a9] -> [a9] = a5 -> b1`
- `b1 = a6 -> b2`
- `a4 = a5 -> b3`
- `a10 -> [a10] -> [a10] = b3 -> b4`
- `a1 -> a2 -> a3 = a4 -> b5`
- `b5 = a6 -> b6`
- `b4 = b6 -> b7`

Example Typing – Current State

```
map f [] = []
```

```
map f (x : xs) = f x : map f xs
```

- setup

- $\text{map} :: a1 \rightarrow a2 \rightarrow a3, \dots$

- constraints

- $U = \{a1 = a4, a2 = [a7], a2 = b2, a3 = [a8], a3 = b7, a9 \rightarrow [a9] \rightarrow [a9] = a5 \rightarrow b1, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 \rightarrow [a10] \rightarrow [a10] = b3 \rightarrow b4, a1 \rightarrow a2 \rightarrow a3 = a4 \rightarrow b5, b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7\}$

- connection of constraints and types via substitution τ , mapping type-variables to types

- theorem: $(s\tau = t\tau \text{ for all } s = t \in U)$ iff $\text{map} :: (a1 \rightarrow a2 \rightarrow a3)\tau$
 - task: find most general τ such $s\tau = t\tau$ for all $s = t \in U$ unification problem
 - such a most general unifier (mgu) τ yields the most general type for map
 - unification is decidable and a most general unifier can be computed
 - unification is the core algorithm for type-inference
(unification works on terms, and indeed types are terms where $[.]$ is unary symbol, $. \rightarrow .$ is binary symbol, Bool and Int are constants, etc.)

Unification Algorithm of Martelli & Montanari

Transform unification problem U until no further rules are applicable

- $\{s = s\} \uplus U \hookrightarrow U$ (delete)
- $\{f(s_1, \dots, s_n) = f(t_1, \dots, t_n)\} \uplus U \hookrightarrow \{s_1 = t_1, \dots, s_n = t_n\} \cup U$ (decompose)
- $\{f(\dots) = g(\dots)\} \uplus U \hookrightarrow \perp$, if $f \neq g$ (clash)
- $\{f(\dots) = x\} \uplus U \hookrightarrow \{x = f(\dots)\} \cup U$ (swap)
- $\{x = t\} \uplus U \hookrightarrow \{x = t\} \cup U\{x/t\}$, if $x \in \text{Vars}(U) \setminus \text{Vars}(t)$ (substitute)
- $\{x = t\} \uplus U \hookrightarrow \perp$, if $x \in \text{Vars}(t)$ and $x \neq t$ (occurs check)

Properties

- $\hookrightarrow$ terminates
- if $U \hookrightarrow V$ then U and V have the same unifiers ($\perp$ has no unifiers)
- if $U \hookrightarrow^! V$ ($U \hookrightarrow^* V$ and there is no $\hookrightarrow$ -step possible on V) then either
 - $V = \perp$ and U has no unifier, or
 - $V = \{x_1 = t_1, \dots, x_n = t_n\}$ encodes a substitution τ , where the list $x_1, \dots, x_n$ is distinct and $\{x_1, \dots, x_n\} \cap (\text{Vars}(t_1) \cup \dots \cup \text{Vars}(t_n)) = \emptyset$; moreover τ is an mgu of U

Example: Unification to Determine Type of Map

- $a1 = a4, a2 = [a7], a2 = b2, a3 = [a8], a3 = b7,$
 $a9 \rightarrow [a9] \rightarrow [a9] = a5 \rightarrow b1, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3,$
 $a10 \rightarrow [a10] \rightarrow [a10] = b3 \rightarrow b4, a1 \rightarrow a2 \rightarrow a3 = a4 \rightarrow b5,$
 $b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7$
- decompose: $a1 = a4, \underline{a2 = [a7]}, a2 = b2, a3 = [a8], a3 = b7, a9 = a5,$
 $[a9] \rightarrow [a9] = b1, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3,$
 $[a10] \rightarrow [a10] = b4, a2 \rightarrow a3 = b5, b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7$
- substitute: $a1 = a4, a2 = [a7], [a7] = b2, \underline{a3 = [a8]}, a3 = b7, a9 = a5,$
 $[a9] \rightarrow [a9] = b1, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3,$
 $[a10] \rightarrow [a10] = b4, [a7] \rightarrow a3 = b5, b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7$
- substitute: $a1 = a4, a2 = [a7], [a7] = b2, a3 = [a8], [a8] = b7, a9 = a5,$
 $[a9] \rightarrow [a9] = b1, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3,$
 $[a10] \rightarrow [a10] = b4, [a7] \rightarrow [a8] = b5, b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7$

Example: Unification to Determine Type of Map

- $a1 = a4, a2 = [a7], [a7] = b2, a3 = [a8], [a8] = b7, a9 = a5,$
 $[a9] \rightarrow [a9] = b1, \underline{b1 = a6 \rightarrow b2}, a4 = a5 \rightarrow b3, a10 = b3,$
 $[a10] \rightarrow [a10] = b4, [a7] \rightarrow [a8] = b5, b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7$
- substitute: $a1 = a4, a2 = [a7], [a7] = b2, a3 = [a8], [a8] = b7, a9 = a5,$
 $[a9] \rightarrow [a9] = a6 \rightarrow b2, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3,$
 $[a10] \rightarrow [a10] = b4, [a7] \rightarrow [a8] = b5, \underline{b5 = a6 \rightarrow b6}, b4 = b6 \rightarrow b7$
- substitute: $a1 = a4, a2 = [a7], [a7] = b2, a3 = [a8], [a8] = b7, a9 = a5,$
 $[a9] \rightarrow [a9] = a6 \rightarrow b2, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3,$
 $[a10] \rightarrow [a10] = b4, [a7] \rightarrow [a8] = a6 \rightarrow b6, b5 = a6 \rightarrow b6, \underline{b4 = b6 \rightarrow b7}$
- substitute: $a1 = a4, a2 = [a7], [a7] = b2, a3 = [a8], [a8] = b7, a9 = a5,$
 $[a9] \rightarrow [a9] = a6 \rightarrow b2, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3,$
 $[a10] \rightarrow [a10] = b6 \rightarrow b7, [a7] \rightarrow [a8] = a6 \rightarrow b6, b5 = a6 \rightarrow b6,$
 $b4 = b6 \rightarrow b7$

Example: Unification to Determine Type of Map

- $a1 = a4, a2 = [a7], [a7] = b2, a3 = [a8], [a8] = b7, a9 = a5,$
 $[a9] \rightarrow [a9] = a6 \rightarrow b2, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3,$
 $[a10] \rightarrow [a10] = b6 \rightarrow b7, [a7] \rightarrow [a8] = a6 \rightarrow b6, b5 = a6 \rightarrow b6,$
 $b4 = b6 \rightarrow b7$
- decompose: $a1 = a4, a2 = [a7], [a7] = b2, a3 = [a8], [a8] = b7, \underline{a9 = a5},$
 $[a9] = a6, [a9] = b2, b1 = a6 \rightarrow b2, \underline{a4 = a5 \rightarrow b3}, \underline{a10 = b3}, [a10] = b6,$
 $[a10] = b7, [a7] = a6, [a8] = b6, b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7$
- substitute: $a1 = a5 \rightarrow b3, a2 = [a7], \underline{[a7] = b2}, a3 = [a8], \underline{[a8] = b7}, a9 = a5,$
 $[a5] = a6, [a5] = b2, b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3, [b3] = b6,$
 $[b3] = b7, [a7] = a6, [a8] = b6, b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7$
- swap: $a1 = a5 \rightarrow b3, a2 = [a7], b2 = [a7], a3 = [a8], b7 = [a8], a9 = a5,$
 $a6 = [a5], b2 = [a5], b1 = a6 \rightarrow b2, a4 = a5 \rightarrow b3, a10 = b3, b6 = [b3],$
 $b7 = [b3], a6 = [a7], b6 = [a8], b5 = a6 \rightarrow b6, b4 = b6 \rightarrow b7$

Example: Unification to Determine Type of Map

- $a1 = a5 \rightarrow b3$, $a2 = [a7]$, $b2 = [a7]$, $a3 = [a8]$, $b7 = [a8]$, $a9 = a5$,
 $a6 = [a5]$, $b2 = [a5]$, $b1 = a6 \rightarrow b2$, $a4 = a5 \rightarrow b3$, $a10 = b3$, $b6 = [b3]$,
 $b7 = [b3]$, $a6 = [a7]$, $b6 = [a8]$, $b5 = a6 \rightarrow b6$, $b4 = b6 \rightarrow b7$
- substitute: $a1 = a5 \rightarrow b3$, $a2 = [a7]$, $b2 = [a7]$, $a3 = [a8]$, $b7 = [a8]$, $a9 = a5$,
 $a6 = [a5]$, $[a7] = [a5]$, $b1 = [a5] \rightarrow [a7]$, $a4 = a5 \rightarrow b3$, $a10 = b3$,
 $b6 = [b3]$, $[a8] = [b3]$, $[a5] = [a7]$, $[b3] = [a8]$, $b5 = [a5] \rightarrow [b3]$,
 $b4 = b6 \rightarrow [a8]$
- decompose: $a1 = a5 \rightarrow b3$, $a2 = [a7]$, $b2 = [a7]$, $a3 = [a8]$, $b7 = [a8]$,
 $a9 = a5$, $a6 = [a5]$, $a7 = a5$, $b1 = [a5] \rightarrow [a7]$, $a4 = a5 \rightarrow b3$, $a10 = b3$,
 $b6 = [b3]$, $a8 = b3$, $a5 = a7$, $b3 = a8$, $b5 = [a5] \rightarrow [b3]$, $b4 = b6 \rightarrow [a8]$
- substitute: $a1 = a5 \rightarrow b3$, $a2 = [a5]$, $b2 = [a5]$, $a3 = [b3]$, $b7 = [b3]$, $a9 = a5$,
 $a6 = [a5]$, $a7 = a5$, $b1 = [a5] \rightarrow [a5]$, $a4 = a5 \rightarrow b3$, $a10 = b3$, $b6 = [b3]$,
 $a8 = b3$, $a5 = a5$, $b3 = b3$, $b5 = [a5] \rightarrow [b3]$, $b4 = [b3] \rightarrow [b3]$
- delete: $a1 = a5 \rightarrow b3$, $a2 = [a5]$, $b2 = [a5]$, $a3 = [b3]$, $b7 = [b3]$, $a9 = a5$,
 $a6 = [a5]$, $a7 = a5$, $b1 = [a5] \rightarrow [a5]$, $a4 = a5 \rightarrow b3$, $a10 = b3$, $b6 = [b3]$,
 $a8 = b3$, $b5 = [a5] \rightarrow [b3]$, $b4 = [b3] \rightarrow [b3]$

Example: Unification to Determine Type of Map

- final result of unification algorithm: mgu τ
 $a1 = a5 \rightarrow b3, a2 = [a5], b2 = [a5], a3 = [b3], b7 = [b3], a9 = a5,$
 $a6 = [a5], a7 = a5, b1 = [a5] \rightarrow [a5], a4 = a5 \rightarrow b3, a10 = b3, b6 = [b3],$
 $a8 = b3, b5 = [a5] \rightarrow [b3], b4 = [b3] \rightarrow [b3]$
- most general type of `map`: $(a1 \rightarrow a2 \rightarrow a3)\tau$, i.e.,
 $(a5 \rightarrow b3) \rightarrow [a5] \rightarrow [b3]$

Remarks

- we introduced fresh variables for every variable, for every argument of the function, and every non-atomic subexpression
 - this provides a **systematic way** (algorithm) to setup constraints
 - when doing type-inference manually, one often immediately sees certain connections and uses less variables and less constraints
- failures when running the unification algorithm correspond to type-errors of Haskell programs
 - clash appears on type-inference for function `f xs = True ++ xs`:
constant `(++)` `:: [a] -> [a] -> [a]`, but first argument `True` `:: Bool`;
this results in clash of equation `[] (a) = Bool`
 - occurs check appears on type-inference for function `f x = x : x`:
subexpression `(:) x` `:: [a] -> [a]`, but the next argument is `x` `:: a`;
this results in occurs check of equation `[a] = a`

Extensions of the Type-Inference System

- extend expressions, e.g., by allowing `let` and `\ x -> e` (exercises)
- integrate type-classes
 - several functions are defined in type-classes or have type-class constraints
 - `fromEnum :: Enum a => a -> Int`
 - `sort :: Ord a => [a] -> [a]`
 - `5 :: Num a => a`
 - these constraints have to be collected in addition to the equalities in the unification algorithm
 - whenever the variables in type-class constraints get instantiated, one needs to look into the type-class instances to check the instantiation
 - examples are given on the next slide, without providing a full algorithm

Extensions of the Type-Inference System

- example 1

- we know `map :: (a -> b) -> [a] -> [b]` and `show :: Show c => c -> String`
- type-inference on `map show` works as follows
 - `map show :: ([a] -> [b]) $\tau$` , for τ being mgu of $U = \{(a \rightarrow b) = (c \rightarrow \text{String})\}$ for constraints $C = \{\text{Show } c\}$
 - $U \hookrightarrow \{a = c, b = \text{String}\}$ and C remains unchanged
 - result: `map show :: Show c => [c] -> [String]` where C is added as constraint

- example 2

- type-inference on `f x = map show [(x, True, 'c')]` works as follows
 - assume `x :: a`
 - `map show :: Show b => [b] -> [String]`
 - `[(x, True, 'c')] :: [(a, Bool, Char)]`
 - unification leads to `b = (a, Bool, Char)`
 - now `Show b` is instantiated to `Show (a, Bool, Char)` and simplified to `Show a`, since
 - `instance (Show a, Show b, Show c) => Show (a, b, c)`
 - `instance Show Bool`
 - `instance Show Char`
 - result: `f :: Show a => a -> [String]`

Limits of Type-Inference in the Presence of Type-Classes

- consider `f = if 2 * 2^62 < 0 then "overflow" else "okay"`
 - question: which number-type is chosen for the comparison?
`Int` or `Integer` or `Float` or `Double`
 - type-inference is of no help, e.g., `(<) (2 * 2^62) :: (Num a, Ord a) => a -> Bool`,
i.e., `2 * 2^62 < 0 :: Bool` for any suitable `a`
- default rule
 - for numeric types, Haskell uses a default rule: choose `Integer` as default, or switch to `Double` if fractional computations are involved (`2.0 < 4`)
 - if one does not want to use default types, provide explicit type annotation
 - note: defaults can be overwritten, e.g. by line `default (Int, Float)`
- examples
 - `f` evaluates to `"okay"`
 - `g = if 2 * 2^62 < (0 :: Int) then "overflow" else "okay"` yields `"overflow"`
 - `[]` in ghci is `show ([] :: [a])` which evaluates to string `[]` after defaulting `a` to `Integer`
 - `[] :: String` in ghci is `show ([] :: String)` which evaluates to string `" "`

Limits of Default Rule

- built-in default rule is restricted to built-in numeric type classes

- consider function definition

```
f :: String -> Bool
```

```
f xs = show (read xs) == xs
```

- function `f` takes input `xs`, parses it into an element, which is then converted back to a string via `show` and compared to the input

- `read xs :: Read a => a`

- `show (read xs) :: (Show a, Read a) => String`

where `a` is the type for the intermediate result of `read xs`

- it is completely unclear, which type `a` should be: `Int`, `Bool`, `[Double]`, ...

- ghc complains about ambiguous type variables at this point

- solution: provide explicit type annotation, e.g.

```
f xs = show (read xs :: [(Int, Bool)]) == xs
```

Literature

- Simon Thompson, The Craft of Functional Programming, Second Edition, Addison–Wesley, Chapter 13: “Checking Types”
- J. Roger Hindley. The Principal Type-Scheme of an Object in Combinatory Logic. Transactions of the American Mathematical Society, volume 146, pages 29—60.
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