universität innsbruck

Automata and Logic | 25W | LVA 703302

1st EXAM | February 2, 2026

This exam consists of **four** exercises. The available points for each item are written in the margin. ***Explain your answers!***

1 Determine whether the following sets over $\Sigma = \{a, b\}$ are regular or not. Prove your answers.

⟨10⟩ (a) $\{xay \mid x, y \in \{a\}^* \text{ and } x \neq y\}$

⟨10⟩ (b) $\{xy \mid x, y \in \Sigma^* \text{ and } x = y\} \cup \{xy \mid x, y \in \Sigma^* \text{ and } x \neq y\}$

2

⟨15⟩ (a) Construct a WMSO formula for the set $\{x \in \{a, b\}^* \mid |x| \text{ is even}\}$.

⟨15⟩ (b) Consider the WMSO formula $\varphi = \exists y.\, y < x \vee Y(y)$. Give automata or regular expressions for the atomic subformulas and explain the operations needed to obtain the regularity of $L_a(\varphi)$.

3

⟨10⟩ (a) Consider the alternating finite automaton $M = (\{p, q, r\}, \{a, b\}, \Delta, p, \{r\})$ with

$$\Delta(p, a) = p \wedge q \qquad \Delta(q, a) = \top \qquad \Delta(r, a) = p$$
$$\Delta(p, b) = r \qquad \Delta(q, b) = p \vee r \qquad \Delta(r, b) = \bot$$

Which of the following strings belong to $L(M)$?

i. *bbab* ii. *abab* iii. *aaba*

⟨20⟩ (b) Construct an alternating Büchi automaton M that accepts the set

$$\{x \in \{a, b\}^\omega \mid x \text{ contains } aaa \text{ and } bb \text{ as substring}\}$$

and show an accepting run of M on the infinite string $baabaaab^\omega$.

⟨20⟩ 4 Determine whether the following statements are true or false. (Providing explanation is optional.) Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

1. $\neg(\varphi \,\mathsf{U}\, \psi) \equiv \neg\varphi \,\mathsf{R}\, \neg\psi$
2. Every ω-regular set is accepted by a DBA.
3. Every GBA can be transformed into equivalent NBA.
4. A nondeterministic finite automaton may have multiple start states.
5. For DFAs M and N, the question whether $L(M) = L(N)$ is undecidable.
6. A set $A \subseteq \Sigma^*$ is regular if and only if $A = L(\varphi)$ for some WMSO formula φ.
7. Regular sets are effectively closed under intersection, union and complement.
8. The set $\{x \in \{a, b\}^\omega \mid x \text{ contains } a, aba \text{ and } aabaa \text{ as substring}\}$ is ω-regular.
9. For a generalized Büchi automata G, the question whether $L(G) = \varnothing$ is decidable.
10. Every alternating Büchi automaton can be effectively transformed into an equivalent nondeterministic Büchi automaton.