1st EXAM – SOLUTIONS February 2, 2026

[1] (a) The set $A = \{xay \mid x, y \in \{a\}^* \text{ and } x \neq y\}$ is regular since it consists of all strings over $\{a\}^*$ having at least two $a$'s, so $A = L(aaa^*)$.

(b) The set $\{xy \mid x, y \in \Sigma^* \text{ and } x \neq y\}$ consists of all non-empty strings over $\Sigma$ since we can take $x = \epsilon$. Since the empty string belongs to $\{xy \mid x, y \in \Sigma^* \text{ and } x = y\}$, it follows that the set $B$ consists of all strings over $\Sigma$, which is obviously regular.

[2] (a)
$$\begin{aligned}\psi \;=\; & \exists\, \ell.\, \neg P_a(\ell) \wedge \neg P_b(\ell) \wedge \big(\forall\, x.\, \neg P_a(x) \wedge \neg P_b(x) \rightarrow \ell \leqslant x\big) \\ & \wedge \exists\, X.\, X(0) \wedge \big(\forall\, x.\, \forall\, y.\, y = x + 1 \wedge y \leqslant \ell \rightarrow (X(x) \leftrightarrow \neg X(y))\big) \wedge X(\ell)\end{aligned}$$

(b) First consider the subformula $\psi\colon\ y < x \vee Y(y)$ with $\mathsf{FV}(\psi) = (x, y, Y)$. We have

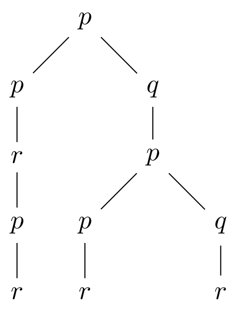
$$\begin{aligned} L_a(y < x) &= \tbinom{0}{0}^* \tbinom{0}{1} \tbinom{0}{0}^* \tbinom{1}{0} \tbinom{0}{0}^* \\ L_a(Y(y)) &= \Big[\tbinom{0}{0} + \tbinom{0}{1}\Big]^* \tbinom{1}{1} \Big[\tbinom{0}{0} + \tbinom{0}{1}\Big]^* \end{aligned}$$

and compute

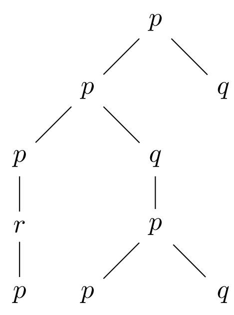
$$\begin{aligned} L_1 &= \mathsf{drop}_3^{-1}(L_a(y < x)) = \begin{pmatrix}0\\0\\*\end{pmatrix}^* \begin{pmatrix}0\\1\\*\end{pmatrix} \begin{pmatrix}0\\0\\*\end{pmatrix}^* \begin{pmatrix}1\\0\\*\end{pmatrix} \begin{pmatrix}0\\0\\*\end{pmatrix}^* \\ L_2 &= \mathsf{drop}_1^{-1}(L_a(Y(y))) = \left[\begin{pmatrix}*\\0\\0\end{pmatrix} + \begin{pmatrix}*\\0\\1\end{pmatrix}\right]^* \begin{pmatrix}*\\1\\1\end{pmatrix} \left[\begin{pmatrix}*\\0\\0\end{pmatrix} + \begin{pmatrix}*\\0\\1\end{pmatrix}\right]^* \end{aligned}$$

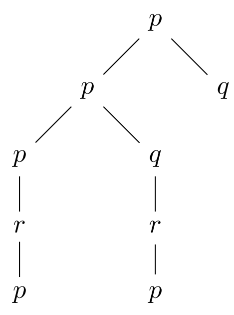
We have $L_a(\psi) = (L_1 \cup L_2) \cap L(\mathcal{A}_{2,1})$. Here $\mathcal{A}_{2,1}$ is a DFA to check the admissibility condition. For $\varphi$ we obtain $L_a(\varphi) = \mathsf{stz}(\mathsf{drop}_2(L_a(\psi)))$.

[3] (a) The string $bbab$ does not belong to $L(M)$ because after the first $b$ the automaton is in state $r$ but $\Delta(r, b) = \bot$. The string $abab$ belongs to $L(M)$ because of the accepting run




The string $aaba$ does not belong to $L(M)$ because the only runs



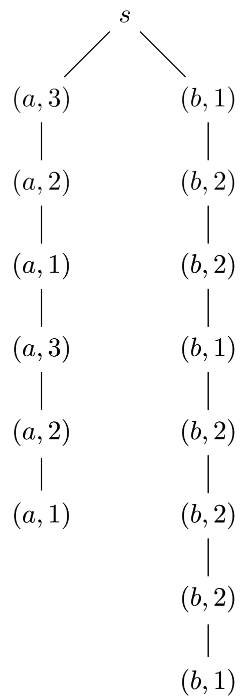



are not accepting.

(b) The ABA $M = (Q, \{a, b\}, \Delta, s, \varnothing)$ with $Q = \{s\} \cup \{a\} \times \{1, 2, 3\} \cup \{b\} \times \{1, 2\}$ and

$$\begin{aligned}
\Delta(s, a) &= (a, 2) \wedge (b, 2) \\
\Delta(s, b) &= (a, 3) \wedge (b, 1) \\
\Delta((a, 3), a) &= (a, 2) \\
\Delta((a, 2), a) &= (a, 1) \\
\Delta((a, 1), a) &= \top \\
\Delta((a, 3), b) &= \Delta((a, 2), b)) = \Delta((a, 1), b) = (a, 3) \\
\Delta((b, 2), b) &= (b, 1) \\
\Delta((b, 1), b) &= \top \\
\Delta((b, 2), a) &= \Delta((b, 1), a)) = (b, 2)
\end{aligned}$$

accepts the given set. The following is an accepting run of $M$ on the string $baabaaab^\omega$:

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1. True.
2. False.
3. True.
4. True.
5. False.
6. True.
7. True.
8. True.
9. True.
10. True.