Automata and Logic	25W	LVA 703026 + 703027
Lecture 1		October 10, 2025

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on October 10.

Exercises

⟨2⟩ 1. Design DFAs for the following sets.

   (a) The set of strings in $\{a, b, c\}^*$ containing the substring *cab*.

   (b) The set of strings $x \in \{0, 1, 2\}^*$ that are ternary representations, leading zeros permitted, of numbers that are not multiples of four. (The empty string represents zero.)

⟨2⟩ 2. Let $M = (Q, \Sigma, \delta, s, F)$ be an arbitrary DFA. Prove by induction on $|y|$ that

$$\widehat{\delta}(q, xy) = \widehat{\delta}(\widehat{\delta}(q, x), y)$$

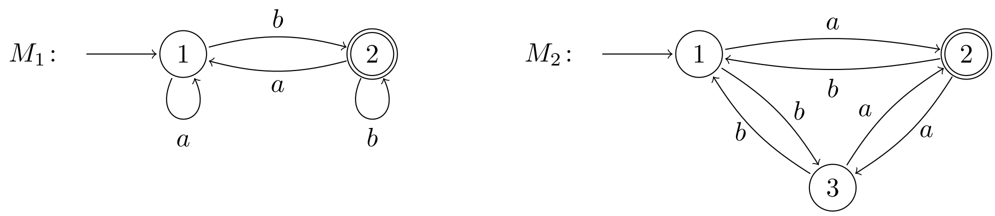
for all strings $x, y \in \Sigma^*$ and states $q \in Q$.

⟨3⟩ 3. A set $A \subseteq \Sigma^*$ is said to be reflexive if $\epsilon \in A$ and transitive if $AA \subseteq A$. Prove that A^* is the smallest reflexive and transitive set containing A, for any $A \subseteq \Sigma^*$.

⟨3⟩ 4. (a) Prove that regular sets are effectively closed under symmetric difference ($\triangle$) defined as:

$$A \triangle B = \{x \mid x \in A \text{ or } x \in B \text{ but not } x \in A \cap B\}$$

   (b) Consider the DFAs



Construct a DFA M such that $L(M) = L(M_1) \triangle L(M_2)$.