universität innsbruck

Automata and Logic	25W	LVA 703026 + 703027
Lecture 2 + 3		October 24, 2025

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on October 24.

Exercises

⟨2⟩ 1. Construct NFAs for the following sets. Try to take advantage of nondeterminism as much as possible.
   (a) The set of strings over $\Sigma = \{0, 1, \ldots, 9\}$ such that the final digit has appeared before.
   (b) The set of non-empty strings over $\Sigma = \{0, 1, \ldots, 9\}$ such that the final digit has *not* appeared before.

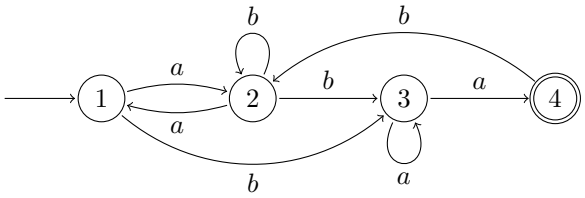
⟨2⟩ 2. Consider the following NFA$_\epsilon$ N:

		ϵ	a	b	c
$\to$	1	$\varnothing$	$\{1\}$	$\{2\}$	$\{3\}$
	2	$\{1\}$	$\{2\}$	$\{3\}$	$\varnothing$
	$3F$	$\{2\}$	$\{3\}$	$\varnothing$	$\{1\}$

   (a) Compute the ϵ-closure of each state.
   (b) List all strings in $L(N)$ of length two or less.
   (c) Transform N into an equivalent DFA without inaccessible states.

⟨2⟩ 3. Give regular expressions for each of the following subsets of $\{a, b, c\}^*$:
   (a) $\{x \mid x$ contains an even number of b's$\}$
   (b) $\{x \mid x$ contains an even number of a's or an odd number of b's$\}$

⟨2⟩ 4. Using the method on slide 10 of lecture 3, transform the following automaton into an equivalent regular expression:



⟨2⟩ 5. (a) Prove that regular sets are closed under string reversal.
   (b) Prove that every regular set is accepted by an NFA with exactly one final state.

Bonus Exercise

⟨5⟩ 6. Given a set A of strings, define the set $\mathrm{MT}(A) = \{y \mid xyz \in A$ and $|x| = |y| = |z|$ for some strings x and $y\}$.
   (a) Prove that $\mathrm{MT}(A)$ is regular whenever A is regular.
   (b) Does the converse hold?