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Automata and Logic	25W	LVA 703026 + 703027
Lecture 4		October 31, 2025

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on October 31.

Exercises

⟨2⟩ 1. Consider the following DFA M:

	a	b
$\rightarrow$ 1	1	4
2	3	1
$3F$	4	2
$4F$	3	5

	a	b
5	4	6
6	6	3
7	2	4

(a) Determine which states are accessible.

(b) Compute the equivalence classes of the indistinguishability relation $\approx$.

(c) Compute the minimum-state DFA for $L(M)$.

⟨2⟩ 2. Let $M_\cap$ be the DFA obtained by applying the product construction on slide 25 of lecture 1 to *minimum-state* DFAs M_1 and M_2.

(a) Show that $M_\cap$ may contain inaccessible states.

(b) Let $M'_\cap$ be obtained from $M_\cap$ by removing all inaccessible states. Is $M'_\cap$ minimal? Prove your answer.

⟨2⟩ 3. Consider the set $L = \{a^n b^n \mid n \geqslant 0\}$.

(a) For $A = \{a^n b^m c^k \mid n + m = k\}$ find a homomorphism h_A such that $h_A(A) = L$ or $h_A^{-1}(A) = L$.

(b) For $B = \{a^n b^m \mid 2n = m\}$ find a homomorphism h_B such that $h_B(B) = L$ or $h_B^{-1}(B) = L$.

(c) Assuming that L is not regular,[1] why does the existence of the homomorphisms h_A and h_B show that A and B are not regular?

⟨2⟩ 4. For each WMSO formula φ_i below either give a model α_i such that $\alpha_i \vDash \varphi_i$ and determine the size of α_i, or explain why φ_i is unsatisfiable:

$$\begin{aligned}
\varphi_1 &= \exists\, x.\, X(x) \wedge \exists\, y.\, X(y) \wedge x \neq y \\
\varphi_2 &= (\forall\, x.\, x = 0 \rightarrow X(x)) \wedge (\forall\, x.\, X(x) \rightarrow z < x) \\
\varphi_3 &= \varphi_1 \wedge \forall\, x.\, X(x) \rightarrow \exists\, y.\, Y(y) \wedge x < y
\end{aligned}$$

⟨2⟩ 5. Consider the WMSO formulas

$$\begin{aligned}
\psi_1 &= \exists\, x.\, x = 0 \wedge \forall\, y.\, x < y \rightarrow \neg P_a(y) \wedge \neg P_b(y) \\
\psi_2 &= \forall\, x.\, P_b(x) \rightarrow \exists\, y.\, P_a(y) \wedge x < y \wedge \neg \exists\, z.\, x < z \wedge z < y \\
\psi_3 &= \exists\, x.\, P_a(x) \wedge \forall\, y.\, x \neq y \rightarrow \neg P_a(y)
\end{aligned}$$

and the alphabet $\Sigma = \{a, b\}$. For each $i \in \{1, 2, 3\}$ find a string $x_i \in \Sigma^*$ such that $x_i \in L(\psi_i)$ and $x_i \notin L(\psi_j)$ for all $1 \leqslant j \leqslant 3$ with $i \neq j$.

[1] In lecture 5 this will be proved.