

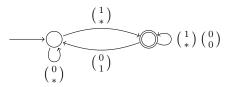
Automata and Logic	25W	LVA 703026 + 703027
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Lecture 6 November 14, 2025

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on November 14.

## **Exercises**

- (3) 1. (a) Construct a WMSO formula for the set  $\{a^mb^n \mid m \text{ is even or } n \text{ is odd}\}$ .
  - (b) Consider the WMSO formula  $\varphi = \exists X. \exists y. x < y \lor X(y)$ . Give automata or regular expressions for the atomic subformulas and explain the operations needed to obtain the regularity of  $L_a(\varphi)$ .
- (2) 2. Read Sections 2.3, 2.4, the paragraph on "Issues in the classical approach" (page 22) and Section 3.2 of the MONA manual. Run MONA on the WMSO formula  $\psi = X(x) \land \exists y. x < y \land X(y)$ . Turn on the automaton output and explain the result in detail. How does MONA's automaton compare to the regular expression for  $L_a(\psi)$  given on slides?
- 3. Consider the following automaton which accepts  $L_a(\varphi)$  for  $\varphi = \exists x. P_a(x) \land \forall y. x < y \rightarrow \neg P_a(y) \land \neg P_b(y)$ .



Compute an automaton for  $L(\varphi)$  according to the lemma on slide 21 of lecture 6 (here  $\Sigma = \{a, b\}$ ). Clearly state what the homomorphism h looks like.

4. Compute a minimal solution for the following puzzle with MONA.

A farmer has to take a dog, a cat, and a mouse across a river using her boat. She can only take one of the animals across at a time. If she takes the mouse, the dog will eat the cat. If she takes the dog, the cat will eat the mouse. How does she get the three animals across the river without hurting anyone?

<sup>1</sup>https://www.brics.dk/mona/mona14.pdf