Automata and Logic | 25W | LVA 703026 + 703027

Lecture 7 | November 21, 2025

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. Solutions for bonus exercises must be submitted separately. The (strict) deadline is 7 am on November 21.

Exercises

⟨3⟩ 1. Consider the Presburger arithmetic formula $\varphi\colon (\exists\, y.\, x = 4y + 3) \vee x = 5$.

(a) Which of the following strings belong to $L(\varphi)$?

i. 11010 ii. 00101 iii. 11101

(b) Construct a finite automaton that accepts $L(x = 4y + 3)$.

(c) Construct a finite automaton that accepts $L(\varphi)$.

⟨3⟩ 2. Prove the second part of the theorem on slide 21, i.e., show the following: A string x is accepted by the automaton A_φ if and only if $\underline{x}$ is a solution for the equation $a_1x_1 + \cdots + a_nx_n = b$.

⟨2⟩ 3. Consider the Presburger arithmetic formula $\varphi\colon \neg\exists\, y.\, x - 3y = 1$.

(a) Which of the following strings belong to $L(\varphi)$?

i. 00011 ii. 10101 iii. 11001

(b) Construct a finite automaton that accepts $L(x - 3y = 1)$

(c) Construct a finite automaton that accepts $L(\varphi)$.

⟨2⟩ 4. Adapt the construction on slide 21 such that A_φ accepts representations of solutions for a given *inequality* $a_1x_1 + \cdots + a_nx_n \leqslant b$. Illustrate your algorithm on the inequality $3x - 2y \leqslant 1$.

Bonus Exercise

⟨5⟩ 5. Let A be a regular set. Consider the sets

$$\begin{aligned} B &= \{x \mid y = x^n \text{ for some } y \in A \text{ and } n \geqslant 0\} \\ C &= \{x \mid x = y^n \text{ for some } y \in A \text{ and } n \geqslant 0\} \end{aligned}$$

One is necessarily regular and one is not. Which is which? Give a proof and a counterexample.