universität innsbruck

Automata and Logic	25W	LVA 703026 + 703027
Lecture 11		January 9, 2026

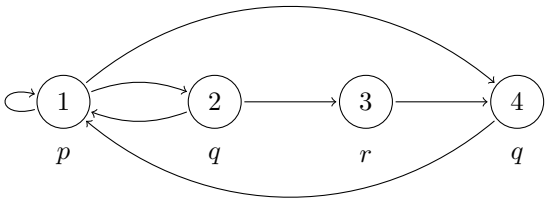
Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on January 9.

Exercises

⟨3⟩ 1. Consider the language $L = \{x \in \{a,b\}^\omega \mid |x|_a = \infty \text{ and } |x|_b = \infty\}$ from slide 9.
   (a) Show that no DBA with only two states accepts L.
   (b) Give a minimal deterministic generalized Büchi automaton (GBA) accepting L.
   (c) Apply the construction on slide 11 to transform your GBA of (b) into an NBA.

⟨2⟩ 2. Determine which of the following equivalences hold. For every equivalence that does not hold give a counterexample.
   (a) $\mathrm{G}\,\varphi \equiv \mathrm{X}\,\mathrm{G}\,\varphi$
   (b) $\mathrm{F}\,\neg(\varphi\,\mathrm{R}\,\psi) \equiv \neg\,\mathrm{G}\,\varphi\,\mathrm{U}\,\neg\,\mathrm{G}\,\psi$
   (c) $(\varphi \to \psi)\,\mathrm{W}\,\chi \equiv \neg\varphi\,\mathrm{W}\,\chi \vee \psi\,\mathrm{W}\,\chi$

⟨2⟩ 3. Consider the following model $\mathcal{M}$:



   (a) For each of the LTL formulas $\phi_1 = \mathrm{X}(p\,\mathrm{U}\,q)$, $\phi_2 = \mathrm{G}\,\mathrm{F}\,p \wedge \mathrm{G}\,\mathrm{F}\,q \wedge \mathrm{G}\,\mathrm{F}\,r$, and $\phi_3 = \mathrm{G}(\mathrm{X}\,r \to q)$, find a path π_i starting at state 1 such that $\pi_i \models \phi_i$ for each $i \in \{1,2,3\}$.
   (b) Give an LTL formula that distinguishes states 2 and 4 of $\mathcal{M}$.

⟨3⟩ 4. Let $\mathcal{V} \in \{\mathrm{F},\mathrm{G}\}^*$ be a sequence consisting of the temporal operators F and G. Show that $\mathrm{F}\,\mathrm{G}\,\varphi \equiv \mathcal{V}\,\mathrm{F}\,\mathrm{G}\,\varphi$ and $\mathrm{G}\,\mathrm{F}\,\varphi \equiv \mathcal{V}\,\mathrm{G}\,\mathrm{F}\,\varphi$.

Bonus Exercise

⟨5⟩ 5. Implement an algorithm that decides whether a given (hardcoded) DBA accepts a specific word $u \cdot v^\omega$, where $u, v \in \Sigma^*$. The submission should include the source code and a short description of how to run it.