universität innsbruck

Automata and Logic | 25W | LVA 703026 + 703027

Lecture 12 | January 16, 2026

Solved exercises must be marked and solutions (as a single PDF file) uploaded in OLAT. The (strict) deadline is 7 am on January 16.

Exercises

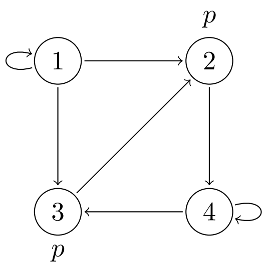
⟨2⟩ 1. Let $\mathsf{AP} = \{r, y, g\}$ be the phases of a traffic light: **r**ed, **y**ellow, and **g**reen. Give the requested specifications in LTL and provide example sequences in $(2^{\mathsf{AP}})^\omega$ that satisfy and violate the formulas.

   (a) Assuming that each phase lasts exactly one discrete unit of time, specify the phase changes of a traffic light. Do not use U, W, or R.

   (b) Now, we drop the assumption that each phase lasts exactly one discrete time unit. Specify the correct phase changes of two independent traffic lights using $\mathsf{AP} = \{r_1, r_2, y_1, y_2, g_1, g_2\}$. Can you reuse your strategy from part (a)? Does your solution scale up to k traffic lights?

⟨3⟩ 2. Consider the LTL formula $\varphi = p \,\mathrm{R}\, (\neg\, \mathrm{X}\, p)$ and the model

$\mathcal{M}$:



   (a) Construct a GBA $A_{\neg\varphi}$ accepting $L(\neg\varphi)$.

   (b) Construct the GBA $A_{\mathcal{M},1}$.

   (c) Construct a GBA $A_{\neg\varphi} \times A_{\mathcal{M},1}$ for the intersection $L(A_{\neg\varphi}) \cap L(A_{\mathcal{M},1})$.

   (d) Check whether $L(A_{\neg\varphi} \times A_{\mathcal{M},1}) = \varnothing$ and hence $\mathcal{M}, 1 \nvDash \varphi$. Provide the corresponding counterexample if $\mathcal{M}, 1 \nvDash \varphi$.

⟨3⟩ 3. Consider the LTL formula $\varphi = (\mathrm{X}\, p) \,\mathrm{U}\, q \wedge (\neg p \wedge q)$.

   (a) Write down the set $\mathcal{C}(\varphi)$.

   (b) Find an elementary set B which contains the subformula φ.

   (c) Give all successor states of the state B in A_φ.

⟨2⟩ 4. Extend the translation from LTL to Büchi automata to directly deal with the F and G connectives. *Hint:* Adapt the definition of elementary set (slide 13) as well as the transition function $\Delta(B, A)$ and acceptance condition F in the definition of A_φ (slide 14).