



Automata and Logic

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Outline

- 1. Summary of Previous Lecture**
- 2. LTL Model Checking**
- 3. Intermezzo**
- 4. LTL Model Checking**
- 5. Further Reading**

Definition

- ▶ **generalized Büchi automaton (GBA)** is automaton $M = (Q, \Sigma, \Delta, S, \{F_1, \dots, F_k\})$ with $F_1, \dots, F_k \subseteq Q$
- ▶ $\infty(r) = \{q \in Q \mid q \text{ occurs infinitely often in run } r\}$
- ▶ run r of GBA is accepting if $\infty(r) \cap F_i \neq \emptyset$ for all $1 \leq i \leq k$

Remark

NBA $(Q, \Sigma, \Delta, S, F)$ is equivalent to GBA $(Q, \Sigma, \Delta, S, \{F\})$

Lemma

every GBA can be transformed into equivalent NBA

Remark

minimal DBAs are not unique

Definition

LTL (linear-time temporal logic) formulas are built from propositional atoms, logical connectives and temporal connectives X , F , G , U , W , R , according to following grammar:

$$\varphi ::= \perp \mid \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \mid X\varphi \mid F\varphi \mid G\varphi \mid \varphi U \varphi \mid \varphi W \varphi \mid \varphi R \varphi$$

Definitions

- ▶ **transition system (model)** is triple $\mathcal{M} = (S, \rightarrow, L)$ with
 - ① set of states S
 - ② transition relation $\rightarrow \subseteq S \times S$ such that $\forall s \in S \exists t \in S$ with $s \rightarrow t$ ("no deadlock")
 - ③ labeling function $L: S \rightarrow 2^{\text{atoms}}$
- ▶ **path** in model $\mathcal{M}$ is infinite sequence $s_1 \rightarrow s_2 \rightarrow \dots$
- ▶ $\forall$ paths $\pi = s_1 \rightarrow s_2 \rightarrow \dots \quad \forall i \geq 1 \quad \pi^i = s_i \rightarrow s_{i+1} \rightarrow \dots$

Definition

satisfaction of LTL formula φ ($\pi \models \varphi$) with respect to path $\pi = s_1 \rightarrow s_2 \rightarrow \dots$ in model $\mathcal{M} = (S, \rightarrow, L)$ is defined by induction on φ :

$\pi \models \top$		$\pi \not\models \perp$		$\pi \models \varphi \wedge \psi \iff \pi \models \varphi \text{ and } \pi \models \psi$
$\pi \models p$	$\iff$	$p \in L(s_1)$		$\pi \models \varphi \vee \psi \iff \pi \models \varphi \text{ or } \pi \models \psi$
$\pi \models \neg \varphi$	$\iff$	$\pi \not\models \varphi$		$\pi \models \varphi \rightarrow \psi \iff \pi \not\models \varphi \text{ or } \pi \models \psi$
$\pi \models X\varphi$	$\iff$	$\pi^2 \models \varphi$		$\pi \models F\varphi \iff \exists i \geq 1 \pi^i \models \varphi$
				$\pi \models G\varphi \iff \forall i \geq 1 \pi^i \models \varphi$
$\pi \models \varphi U \psi$	$\iff$	$\exists i \geq 1 \pi^i \models \psi \text{ and } \forall j < i \pi^j \models \varphi$		
$\pi \models \varphi W \psi$	$\iff$	$(\exists i \geq 1 \pi^i \models \psi \text{ and } \forall j < i \pi^j \models \varphi) \text{ or } \forall i \geq 1 \pi^i \models \varphi$		
$\pi \models \varphi R \psi$	$\iff$	$(\exists i \geq 1 \pi^i \models \varphi \text{ and } \forall j \leq i \pi^j \models \psi) \text{ or } \forall i \geq 1 \pi^i \models \psi$		

Definitions

- ▶ model $\mathcal{M} = (S, \rightarrow, L)$, state $s \in S$, LTL formula φ

$\mathcal{M}, s \models \varphi \iff \forall \text{ paths } \pi = s \rightarrow \dots \quad \pi \models \varphi$ "formula φ holds in state s of $\mathcal{M}$ "

- ▶ LTL formulas φ and ψ are **semantically equivalent** ($\varphi \equiv \psi$) if $\pi \models \varphi \iff \pi \models \psi$ for all models $\mathcal{M} = (S, \rightarrow, L)$ and paths π in $\mathcal{M}$

Theorem

- ▶

$\neg X\varphi \equiv X\neg\varphi$	$\varphi U \psi \equiv \neg(\neg\psi U (\neg\varphi \wedge \neg\psi)) \wedge F\psi$	
$\neg F\varphi \equiv G\neg\varphi$	$F\varphi \equiv \top U \varphi$	$F(\varphi \vee \psi) \equiv F\varphi \vee F\psi$
$\neg G\varphi \equiv F\neg\varphi$	$G\varphi \equiv \perp R \varphi$	$G(\varphi \wedge \psi) \equiv G\varphi \wedge G\psi$
$\neg(\varphi U \psi) \equiv \neg\varphi R \neg\psi$	$\varphi W \psi \equiv \psi R (\varphi \vee \psi)$	$\varphi U \psi \equiv \varphi W \psi \wedge F\psi$
$\neg(\varphi R \psi) \equiv \neg\varphi U \neg\psi$	$\varphi R \psi \equiv \psi W (\varphi \wedge \psi)$	$\varphi W \psi \equiv \varphi U \psi \vee G\varphi$
- ▶ $\{X, U\}$, $\{X, W\}$ and $\{X, R\}$ are **adequate** sets of temporal connectives for LTL

Automata

- ▶ (deterministic, nondeterministic, alternating) finite automata
- ▶ regular expressions
- ▶ (alternating) **Büchi automata**

Logic

- ▶ (weak) monadic second-order logic
- ▶ Presburger arithmetic
- ▶ **linear-time temporal logic**

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Basic Strategy

$\mathcal{M}, s \models \varphi$?

- ① construct **GBA** $A_{\neg\varphi}$ for $\neg\varphi$
- ② combine $A_{\neg\varphi}$ and $\mathcal{M}, s$ into single automaton $A_{\neg\varphi} \times A_{\mathcal{M}, s}$
- ③ test emptiness of $L(A_{\neg\varphi} \times A_{\mathcal{M}, s})$

Notation

AP is (finite) set of propositional atoms used in $\mathcal{M}$ and φ

Definition

trace is infinite string over alphabet 2^{AP}

Plan

GBA $A_{\neg\varphi}$ accepts all traces that violate φ

formula φ in LTL fragment with U and X as only temporal operators

Definitions

► $L(\varphi) = \{x \in (2^{\text{AP}})^\omega \mid x \models \varphi\}$

► $x \models \varphi$ is defined inductively ($x = x_0 x_1 x_2 \dots$)

$$x \models \top$$

$$x \models p \iff p \in x_0$$

$$x \models \neg \varphi \iff x \not\models \varphi$$

$$x \models \varphi \wedge \psi \iff x \models \varphi \text{ and } x \models \psi$$

$$x \models X\varphi \iff x_1 x_2 \dots \models \varphi$$

$$x \models \varphi \text{ U } \psi \iff \exists i \geq 0 \ (x_i x_{i+1} \dots \models \psi \text{ and } \forall j < i \ x_j x_{j+1} \dots \models \varphi)$$

Plan

GBA A_φ will accept $L(\varphi)$

Basic Idea

- ▶ given infinite string $x = x_0x_1x_2 \cdots \in L(\varphi)$
expand $x_i \subseteq AP$ by subformulas ψ of φ to obtain infinite string $y = y_0y_1y_2 \cdots$ such that

$$\psi \in y_i \iff x_i x_{i+1} x_{i+2} \cdots \models \psi$$

negations of subformulas are also considered

- ▶ $y_0, y_1, \dots$ are states in A_φ
- ▶ transition relation and acceptance conditions of A_φ ensure that y is accepting run for x

Example

$\varphi = a \mathbf{U} (\neg a \wedge b)$ and $x = \{a\}\{a, b\}\{b\} \cdots$

$$y_0 = \{a, \neg b, \neg(\neg a \wedge b), \varphi\} \quad y_1 = \{a, b, \neg(\neg a \wedge b), \varphi\} \quad y_2 = \{\neg a, b, \neg a \wedge b, \varphi\}$$

Definition

closure $\mathcal{C}(\varphi)$ of φ consists of all subformulas of φ and their negations, identifying $\neg\neg\psi$ and ψ

Example

$$\mathcal{C}(a \cup (\neg a \wedge b)) = \{a, b, \neg a, \neg b, \neg a \wedge b, \neg(\neg a \wedge b), a \cup (\neg a \wedge b), \neg(a \cup (\neg a \wedge b))\}$$

- ▶ $\{a, b, \neg a \wedge b, a \cup (\neg a \wedge b)\}$ not elementary
- ▶ $\{a, b, a \cup (\neg a \wedge b)\}$ not elementary
- ▶ $\{a, b, \neg(\neg a \wedge b), a \cup (\neg a \wedge b)\}$ elementary
- ▶ $\{\neg a, \neg b, \neg(\neg a \wedge b), a \cup (\neg a \wedge b)\}$ not elementary
- ▶ $\{a, b, \neg(\neg a \wedge b), \neg(a \cup (\neg a \wedge b))\}$ elementary
- ▶ $\{a, \neg b, \neg(\neg a \wedge b), \neg(a \cup (\neg a \wedge b))\}$ elementary

Definition

set $B \subseteq \mathcal{C}(\varphi)$ is **elementary** if it is

① **consistent with respect to propositional logic**: for all $\varphi_1 \wedge \varphi_2 \in \mathcal{C}(\varphi)$ and $\psi \in \mathcal{C}(\varphi)$

$$\blacktriangleright \varphi_1 \wedge \varphi_2 \in B \iff \varphi_1 \in B \text{ and } \varphi_2 \in B$$

$$\blacktriangleright \psi \in B \implies \neg\psi \notin B$$

$$\blacktriangleright \top \in \mathcal{C}(\varphi) \implies \top \in B$$

② **locally consistent with respect to U**: for all $\varphi_1 \mathbf{U} \varphi_2 \in \mathcal{C}(\varphi)$

$$\blacktriangleright \varphi_2 \in B \implies \varphi_1 \mathbf{U} \varphi_2 \in B$$

$$\blacktriangleright \varphi_1 \mathbf{U} \varphi_2 \in B \text{ and } \varphi_2 \notin B \implies \varphi_1 \in B$$

③ **maximal**: for all $\psi \in \mathcal{C}(\varphi)$

$$\blacktriangleright \psi \notin B \implies \neg\psi \in B$$

Lemma

$$\varphi \mathbf{U} \psi \equiv \psi \vee (\varphi \wedge \mathbf{X}(\varphi \mathbf{U} \psi))$$

Definition

GBA $A_\varphi = (\mathbf{Q}, 2^{\mathbf{AP}}, \mathbf{\Delta}, \mathbf{S}, \mathbf{F})$ for LTL formula φ with atoms in AP:

► $\mathbf{Q} = \{B \subseteq \mathcal{C}(\varphi) \mid B \text{ is elementary}\}$

► $\mathbf{S} = \{B \in \mathbf{Q} \mid \varphi \in B\}$

► $\mathbf{\Delta}(B, A) = \begin{cases} \emptyset & \text{if } A \neq B \cap \mathbf{AP} \\ \{C \mid C \in \mathbf{Q} \text{ and } \dots\} & \text{if } A = B \cap \mathbf{AP} \end{cases}$ with

① for all $\mathbf{X}\psi \in \mathcal{C}(\varphi)$ $\mathbf{X}\psi \in B \iff \psi \in C$

② for all $\varphi_1 \mathbf{U} \varphi_2 \in \mathcal{C}(\varphi)$ $\varphi_1 \mathbf{U} \varphi_2 \in B \iff \varphi_2 \in B$ or both $\varphi_1 \in B$ and $\varphi_1 \mathbf{U} \varphi_2 \in C$

► $\mathbf{F} = \{\{B \in \mathbf{Q} \mid \varphi_1 \mathbf{U} \varphi_2 \notin B \text{ or } \varphi_2 \in B\} \mid \varphi_1 \mathbf{U} \varphi_2 \in \mathcal{C}(\varphi)\}$

Example

$$\varphi = X a$$

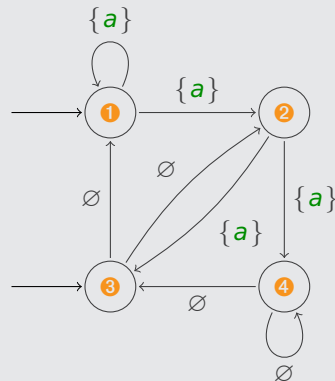
$$\mathcal{C}(\varphi) = \{a, \neg a, X a, \neg X a\}$$

$$\text{states} \quad \textcircled{1} \{a, X a\} \quad \textcircled{2} \{a, \neg X a\} \quad \textcircled{3} \{\neg a, X a\} \quad \textcircled{4} \{\neg a, \neg X a\}$$

$$\text{start states} \quad \textcircled{1} \quad \textcircled{3}$$

transitions	1	2	3	4	
1	✓	✓			$\{a\}$
2			✓	✓	$\{a\}$
3	✓	✓			$\emptyset$
4			✓	✓	$\emptyset$

$$\text{acceptance condition} \quad \emptyset \implies \text{all runs are accepting}$$



Example

$$\varphi = a \cup b$$

$$\mathcal{C}(\varphi) = \{a, \neg a, b, \neg b, a \cup b, \neg(a \cup b)\}$$

$$\text{states} \quad \textcircled{1} \{a, b, \varphi\} \quad \textcircled{2} \{\neg a, b, \varphi\} \quad \textcircled{3} \{a, \neg b, \varphi\} \quad \textcircled{4} \{a, \neg b, \neg \varphi\} \quad \textcircled{5} \{\neg a, \neg b, \neg \varphi\}$$

$$\text{start states} \quad \textcircled{1} \quad \textcircled{2} \quad \textcircled{3}$$

$$\text{transitions}$$

	$\textcircled{1}$	$\textcircled{2}$	$\textcircled{3}$	$\textcircled{4}$	$\textcircled{5}$	
$\textcircled{1}$	✓	✓	✓	✓	✓	$\{a, b\}$
$\textcircled{2}$	✓	✓	✓	✓	✓	$\{b\}$
$\textcircled{3}$	✓	✓	✓			$\{a\}$
$\textcircled{4}$				✓	✓	$\{a\}$
$\textcircled{5}$	✓	✓	✓	✓	✓	$\emptyset$

$$\text{acceptance condition} \quad \{\{\textcircled{1}, \textcircled{2}, \textcircled{4}, \textcircled{5}\}\} \implies \text{runs cycling in state } \textcircled{3} \text{ are not accepting}$$

$$\{a\}^\omega \notin L(A_\varphi) \text{ and } \{b\} \emptyset \{a\}^\omega \in L(A_\varphi)$$

Theorem

$$L(\varphi) \subseteq L(A_\varphi)$$

Proof

- ▶ GBA $A_\varphi = (Q, 2^{\text{AP}}, \Delta, S, F)$
- ▶ let $x = x_0 x_1 x_2 \dots \in L(\varphi)$ and define $y_i = \{\psi \in \mathcal{C}(\varphi) \mid x_i x_{i+1} \dots \models \psi\}$ for all $i \geq 0$
- ▶ $y_0, y_1, y_2, \dots \in Q$ and $y_0 y_1 y_2 \dots$ is accepting **run** for x

$y_{i+1} \in \Delta(y_i, x_i)$: $x_i = y_i \cap \text{AP}$ and for all $X\psi \in \mathcal{C}(\varphi)$

$$X\psi \in y_i \iff x_i x_{i+1} \dots \models X\psi \iff x_{i+1} \dots \models \psi \iff \psi \in y_{i+1}$$

and for all $\varphi_1 \mathbf{U} \varphi_2 \in \mathcal{C}(\varphi)$

$$\varphi_1 \mathbf{U} \varphi_2 \in y_i \iff x_i x_{i+1} \dots \models \varphi_1 \mathbf{U} \varphi_2$$

$$\iff x_i x_{i+1} \dots \models \varphi_2 \text{ or } (x_i x_{i+1} \dots \models \varphi_1 \text{ and } x_{i+1} \dots \models \varphi_1 \mathbf{U} \varphi_2)$$

$$\iff \varphi_2 \in y_i \text{ or } (\varphi_1 \in y_i \text{ and } \varphi_1 \mathbf{U} \varphi_2 \in y_{i+1})$$

Theorem

$$L(\varphi) \subseteq L(A_\varphi)$$

Proof (cont'd)

- ▶ GBA $A_\varphi = (Q, 2^{\text{AP}}, \Delta, S, F)$
- ▶ let $x = x_0 x_1 x_2 \cdots \in L(\varphi)$ and define $y_i = \{\psi \in \mathcal{C}(\varphi) \mid x_i x_{i+1} \cdots \models \psi\}$ for all $i \geq 0$
- ▶ $y_0, y_1, y_2, \dots \in Q$ and $y_0 y_1 y_2 \cdots$ is **accepting** run for x

let $\varphi_1 \cup \varphi_2 \in \mathcal{C}(\varphi)$ and suppose $y_i \notin \{B \in Q \mid \varphi_1 \cup \varphi_2 \notin B \text{ or } \varphi_2 \in B\}$ for some i

$$\begin{aligned} \varphi_1 \cup \varphi_2 \in y_i \text{ and } \varphi_2 \notin y_i &\implies x_i x_{i+1} \cdots \models \varphi_1 \cup \varphi_2 \text{ and } x_i x_{i+1} \cdots \not\models \varphi_2 \\ &\implies x_k x_{k+1} \cdots \models \varphi_2 \text{ for some } k > i \\ &\implies \varphi_2 \in y_k \text{ for some } k > i \\ &\implies y_k \in \{B \in Q \mid \varphi_1 \cup \varphi_2 \notin B \text{ or } \varphi_2 \in B\} \text{ for some } k > i \end{aligned}$$

$y_k \in \{B \in Q \mid \varphi_1 \cup \varphi_2 \notin B \text{ or } \varphi_2 \in B\}$ for infinitely many k

Theorem

$$L(A_\varphi) \subseteq L(\varphi)$$

Proof

- ▶ GBA $A_\varphi = (Q, 2^{\text{AP}}, \Delta, S, F)$
- ▶ let $x = x_0 x_1 x_2 \dots \in L(A_\varphi)$ with accepting run $y = y_0 y_1 y_2 \dots$
- ▶ $\Delta(B, A) = \emptyset$ if $A \neq B \cap \text{AP} \implies x = (y_0 \cap \text{AP})(y_1 \cap \text{AP})(y_2 \cap \text{AP}) \dots$
- ▶ claim: $x \models \varphi$

more general claim: if y is "accepting run" (without requiring y_0 is start state) for x then

$$\psi \in y_0 \iff x \models \psi$$

for all $\psi \in \mathcal{C}(\varphi)$

- ▶ y_0 is start state $\implies \varphi \in y_0 \implies x \models \varphi$

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Question

Which of the following statements hold for $\varphi = (a \wedge X b) \cup \neg a$?

- A** $\neg X a \in C(\varphi)$
- B** $\{b, a \wedge X b, \varphi\}$ is locally consistent with respect to U
- C** $\{\neg a, b, \neg X b, \neg(a \wedge X b), \varphi\} \xrightarrow{\{b\}} \{a, \neg b, X b, a \wedge X b, \neg \varphi\}$ in A_φ
- D** $\{\neg a, b, \neg X b, \neg(a \wedge X b), \varphi\} \xleftarrow{\emptyset} \{a, b, X b, a \wedge X b, \neg \varphi\}$ in A_φ



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Remark

bit vectors can be used to represent 2^{AP} : if $AP = \{a, b, c\}$ then

$$\{a\} \emptyset \{a, c\} \{b, c\} \cdots = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \cdots$$

Basic Strategy

$\mathcal{M}, s \models \varphi$?

- ① construct GBA $A_{\neg\varphi}$ for $\neg\varphi$
- ② combine $A_{\neg\varphi}$ and $\mathcal{M}, s$ into single automaton $A_{\neg\varphi} \times A_{\mathcal{M},s}$
- ③ test emptiness of $L(A_{\neg\varphi} \times A_{\mathcal{M},s})$

Remark

model $\mathcal{M}$ is not GBA

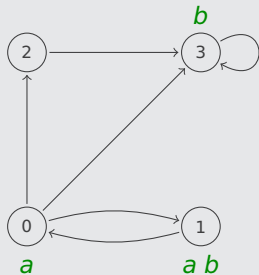
Definition

GBA $A_{\mathcal{M},s} = (S, 2^{\text{AP}}, \Delta, \{s\}, \emptyset)$ for model $\mathcal{M} = (S, \rightarrow, L)$ and state $s \in S$

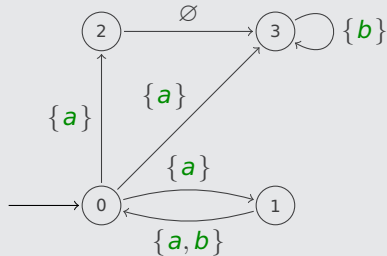
► $\Delta(p, A) = \{q \mid L(p) = A \text{ and } p \rightarrow q\}$

Example

model $\mathcal{M}$



GBA $A_{\mathcal{M},0}$



Intersection of GBAs

- ▶ $A = L(M_1)$ for GBA $M_1 = (Q_1, \Sigma, \Delta_1, S_1, F_1)$ and $B = L(M_2)$ for GBA $M_2 = (Q_2, \Sigma, \Delta_2, S_2, F_2)$
- ▶ $A \cap B = L(M)$ for GBA $M = (Q, \Sigma, \Delta, S, F)$ with
 - ① $Q = Q_1 \times Q_2$
 - ② $S = S_1 \times S_2$
 - ③ $\Delta((p, q), a) = \{(p', q') \mid p' \in \Delta_1(p, a) \text{ and } q' \in \Delta_2(q, a)\}$
 - ④ $F = \{X \times Q_2 \mid X \in F_1\} \cup \{Q_1 \times Y \mid Y \in F_2\}$

Theorem

problem

instance: GBA M

question: $L(M) = \emptyset$?

is decidable

Proof

- ① consider transition graph of GBA $M = (Q, \Sigma, \Delta, S, \{F_1, \dots, F_k\})$
- ② eliminate unreachable states
- ③ compute non-trivial strongly connected components (SCCs) $C_1, \dots, C_n$
- ④ return **no** (**yes**) if there exists (no) SCC C_i such that $C_i \cap F_j \neq \emptyset$ for all $1 \leq j \leq k$

Example

► GBA M with acceptance sets
 $\{0, 9\}$, $\{0, 3, 10\}$, $\{5, 10, 11\}$

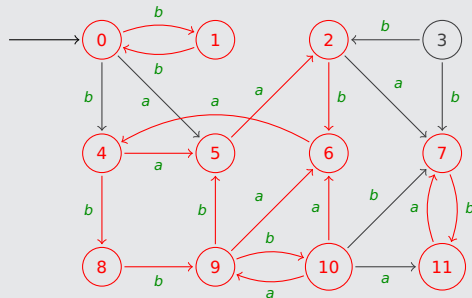
► three SCCs:

1 $C_1 = \{0, 1\}$

2 $C_2 = \{2, 4, 5, 6, 8, 9, 10\}$

3 $C_3 = \{7, 11\}$

► $L(M) \neq \emptyset$



Basic Strategy

$\mathcal{M}, s \models \varphi$?

- ① construct GBA $A_{\neg\varphi}$ for $\neg\varphi$
- ② combine $A_{\neg\varphi}$ and $\mathcal{M}, s$ into single automaton $A_{\neg\varphi} \times A_{\mathcal{M},s}$
- ③ test emptiness of $L(A_{\neg\varphi} \times A_{\mathcal{M},s})$

Theorem

$$\mathcal{M}, s \models \varphi \iff L(A_{\neg\varphi} \times A_{\mathcal{M},s}) = \emptyset$$

Example

GBA $A_{\neg\varphi}$ for $\varphi = a \mathbf{U} b$

	1	2	3	4	5
$\{a, b, \varphi\}$ 1	✓	✓	✓	✓	✓
$\{\neg a, b, \varphi\}$ 2	✓	✓	✓	✓	✓
$\{a, \neg b, \varphi\}$ 3	✓	✓	✓		
$\rightarrow \{a, \neg b, \neg\varphi\}$ 4				✓	✓
$\rightarrow \{\neg a, \neg b, \neg\varphi\}$ 5	✓	✓	✓	✓	✓

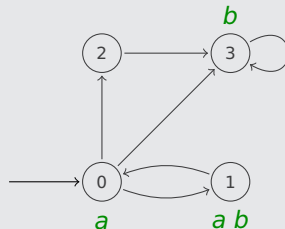
acceptance condition $\{\{1, 2, 4, 5\}\}$

► product automaton $A_{\neg\varphi} \times A_{\mathcal{M}, 0}$

$\rightarrow$ 4 0	{	5 2	}	5 2	{	2 3	}
$\rightarrow$ 5 0	$\emptyset$			2 3	{	2 3	}

► accepting run $4\ 0 \xrightarrow{\{a\}} 5\ 2 \xrightarrow{\emptyset} 2\ 3 \xrightarrow{\{b\}} 2\ 3 \xrightarrow{\{b\}} \dots \Rightarrow \mathcal{M}, 0 \not\models \varphi$

model $\mathcal{M}$



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- ▶ Section 5.2 of **Principles of Model Checking** (MIT Press 2008)

Important Concepts

- ▶ $A_{\mathcal{M},s}$
- ▶ A_φ
- ▶ elementary set

homework for January 16

next lecture (January 19): online evaluation in presence $\implies$ bring device