Exam

This exam consists of <u>four</u> exercises. The available points for each item are written in the margin. You need 50 points to pass. Explain your answers!

[15] **1** Construct ARSs $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ that satisfy the properties indicated in the following table, if possible.

	UN	SN	CR	WN	WCR
$\mathcal{A}_1$	×	×	×	✓	✓
$\mathcal{A}_2$	✓	×	×	×	✓
$\mathcal{A}_3$	✓	×	×	✓	×

2 Consider the TRS $\mathcal{R}$ consisting of the following five rewrite rules:

$$\mathsf{f}(\mathsf{a}) \to \mathsf{a} \qquad \mathsf{g}(\mathsf{a}) \to \mathsf{f}(\mathsf{c}(\mathsf{a}))$$
$$\mathsf{f}(\mathsf{c}(x)) \to \mathsf{c}(\mathsf{c}(\mathsf{f}(x))) \qquad \mathsf{g}(\mathsf{c}(\mathsf{a})) \to \mathsf{f}(\mathsf{g}(\mathsf{a}))$$
$$\mathsf{g}(\mathsf{c}(\mathsf{c}(x))) \to \mathsf{c}(\mathsf{c}(\mathsf{c}(\mathsf{g}(x))))$$

[5] (a) Rewrite the term $\mathsf{g}(\mathsf{c}(\mathsf{f}(\mathsf{a})))$ to normal form.
[5] (b) Is $\mathcal{R}$ confluent?
[10] (c) Prove that $\mathcal{R}$ is polynomially terminating.
[5] (d) Can the termination of $\mathcal{R}$ be shown using LPO?
[10] (e) Can the termination of $\mathcal{R}$ be shown using KBO?

3 Consider the TRS $\mathcal{R} = \{\mathsf{f}(\mathsf{f}(x)) \to \mathsf{g}(\mathsf{g}(\mathsf{h}(x)))\}$.

[5] (a) Prove that $\mathcal{R}$ is terminating.
[10] (b) Compute all critical pairs of $\mathcal{R}$ and determine whether they are convergent.
[10] (c) Construct a complete reduced TRS with the same conversion as $\mathcal{R}$.

4 Consider the TRS combinatory logic

$$\mathsf{S}xyz \to xz(yz) \qquad \mathsf{K}xy \to x \qquad \mathsf{I}x \to x$$

and the term $t = \mathsf{SKI}(\mathsf{KI}(\mathsf{SII}(\mathsf{SII})))$.

[5] (a) Show that t admits an infinite rewrite sequence.
[10] (b) Rewrite t to normal form using the leftmost outermost strategy.
[10] (c) Rewrite t to normal form using the full subsitution strategy.