

# Developing Corpus-based Translation Methods between Informal and Formal Mathematics: Project Description

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**Abstract.** The goal of this project<sup>4</sup> is to (i) accumulate annotated informal/formal mathematical corpora suitable for training semi-automated translation between informal and formal mathematics by statistical machine-translation methods, (ii) to develop such methods oriented at the formalization task, and in particular (iii) to combine such methods with learning-assisted automated reasoning that will serve as a strong semantic component. We describe these ideas, the initial set of corpora, and some initial experiments done over them.

## 1 Introduction and Motivation Ideas

Formal mathematics and automated reasoning are in some sense at the top of the complexity ladder of today's precise ("neat") AI corpora and techniques. Many of us believe that practically all mathematical theorems can be precisely formulated and that their proofs can be written and verified formally, and that this carries over to a lot of the knowledge accumulated by other exact sciences. Given this unmatched expressivity and coverage, automated reasoning over formal mathematics then amounts (or aspires) to being the generic problem-solving technique for arbitrary problems that are expressed in a sufficiently "neat" (formal) language and non-contradictory setting.

The last ten years have brought significant progress in formalization of mathematics and in automated reasoning methods for such formalized corpora. Some graduate textbooks have been formalized, and we have produced general reasoning methods that can often automatically find previous relevant knowledge and prove many smaller steps and lemmas in such textbooks without the necessity to manually provide any further hints or guidance.

However, even routine formalization is today still quite laborious, and the uptake of formalization among mathematicians (and other scientists) is very limited. There is a lot of cognitive processing involved in formalization that is uncommon to majority of today's mathematicians: formalization is a nontrivial skill to learn, and it takes time. As a result, more than 100 years after Turing's birth, most of mathematical (and scientific) knowledge is still largely inaccessible to deep semantic computer processing.

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<sup>4</sup> <http://mws.cs.ru.nl/~mptp/inf2formal>

We believe that this state of affairs can be today helped by automatically *learning* how to formalize (“semanticize”) informal texts, based on the knowledge available in existing large formal corpora. There are several reasons for this belief:

1. Statistical machine learning (data-driven algorithm design) has been responsible for a number of recent AI breakthroughs, such as web search, query answering (IBM Watson), machine translation (Google Translate), image recognition, autonomous car driving, etc. As soon as there are enough data to learn from, data-driven algorithms can automatically learn complicated sets of rules that would be often hard to program and maintain manually.
2. With the recent progress of formalization, reasonably large corpora are emerging that can be (perhaps after additional annotation) used for experiments with machine learning of formalization. The growth of such corpora is only a matter of time, and automated formalization might gradually “bootstrap” this process, making it faster and faster.
3. Statistical machine learning methods have already turned out to be very useful in deductive AI domains such as automated reasoning in large theories (ARLT), thus disproving conjectures that its inherent undecidability makes mathematics into a special field where data-driven techniques cannot apply.
4. Analogously, strong semantic ARLT methods are likely to be useful in the formalization field also for complementing the statistical methods that learn formalization. This could lead to hybrid understanding/thinking AI methods that self-improve on large annotated corpora by cycling between (i) statistical prediction of the text disambiguation based on learning from existing annotations and knowledge, and (ii) improving such knowledge by confirming or rejecting the predictions by the semantic ARLT methods.

The last point (4) is quite unique to the domain of (informal/formal) mathematics, and a good independent reason to start with this AI research. There is hardly any other domain where natural language processing (NLP) could be related to such a firm and expressive semantics as mathematics has, which is additionally to a reasonable degree already checkable with existing ITP and ARLT systems. It is not unimaginable that if we gradually manage to learn how mathematicians (ab)use the normal imprecise vocabulary to convey ideas in the semantically well-grounded mathematical world, such semantic grounding of the natural mathematical language (or at least its underlying mechanisms) will then be also helpful for better semantic treatment of arbitrary natural language texts.

## 2 Approach

The project is in the phase of preparing and analysing suitable corpora, extracting interesting datasets from them on which learning methods can be tried, collecting basic statistics about the corpora. and testing initial learning approaches on them. Initially we consider the following corpora:

**1. The various HOL Light developments:** in particular Flyspeck and Multivariate, for which we have a strong ARLT online service available [2], and which is also in the case of Flyspeck and Multivariate aligned (by Hales) with the informal Flyspeck book. This is the main corpus we have so far worked on. We

have already written programs that collect the links between the informal and formal Flyspeck parts (theorems and definitions), and used such annotations for example for the joint informal/formal HTML presentation of Flyspeck [5]. Currently there are about 250-400 theorems mapped (using the `guid` tag defined by Hales), however we still need to improve our searching mechanism to find all the mapped informal/formal pairs in various parts of the library. In addition to the aligned theorems, Hales has also aligned over 200 concepts, which can be used as the ground level (dictionary) for the statistical translation algorithms. It is likely that further annotation of the texts will be useful, possibly also with some refactoring of the informal and formal parts so that they better correspond to each other. Most of the extraction/alignment chain is now automated so we can update our data after such transformations of the source texts. We export the aligned theorems in several formats: parsed  $\text{\LaTeX}$  via  $\text{\LaTeX}$ ML (using libxml for querying), the original HOL text, bracketed HOL text suitable for parsing into external tools, internal (parsed and type-annotated) representation of the HOL theorems in a Lisp-like notation and in a XML notation, and also representation of each theorem in the (Prolog-parsable) THF TPTP format, containing type declarations of all constants recursively used by the theorems.

**2. The Mizar/MML library:** and in particular its mapping to the book Compendium of Continuous Lattices [1] (CCL) and a smaller mapping to Engelking's General Topology provided by Bancerek.<sup>5</sup> This is a potential large source of informal/formal pairs, however the MML has been developing quickly, and updating the mapping might be necessary to align the books with the current MML for which we have a strong online ARLT service [6,3]. We have also obtained the corresponding  $\text{\LaTeX}$  sources of the CCL book from Cambridge University Press, however we have not yet clarified the possible publication of the data extracted.

**3. The ProofWiki and PlanetMath informal corpora:** We have the XML and  $\text{\LaTeX}$  dumps of these wikis and have used them for initial experiments with disambiguation of informal texts in the student project *Mathifier*,<sup>6</sup> motivated by the NLP work on Wikipedia disambiguation [4]. One relatively surprising preliminary result of this project is quite good performance (75%) of the naive disambiguation algorithm using just the most frequent mathematical meaning without any additional context information. Another initial exploration was done on ProofWiki, whose relatively strict proof style is quite close to the Jaśkowski-style natural deduction used in Mizar. We have measured this by mapping all math expressions and references in the ProofWiki sentences to just one generic expression/reference, and counted the frequency of various proof sentences. The results<sup>7</sup> again show great homogeneity of the corpus, where most of the proof discourse can be superficially mapped to Mizar natural deduction quite economically. Apart from defining and experimenting with such proof-level translation patterns, the main work on these corpora will be their mapping (possibly au-

<sup>5</sup> <http://fm.uwb.edu.pl/mmlquery/fillin.php?filledfilename=t.mqt&argument=number+1>

<sup>6</sup> <http://mws.cs.ru.nl/~urban/Mathifier/>

<sup>7</sup> <http://mizar.cs.ualberta.ca/~mtp/fpk1/opaqcounts1.txt>

tomated by using frequency analysis) to the Mizar and HOL Light corpora, in particular general topology that is developed a lot in ProofWiki and MML.

## 2.1 Methods, Tools and Planned Experiments

There is a lot of relevant NLP research in (i) machine translation (algorithms that directly translate between two languages) (ii) word-sense disambiguation (algorithms that determine the exact meaning of (sets of) words in sentences), and (iii) part-of-speech tagging and phrasal and dependency parsing . The most successful statistical methods (e.g., n-gram-based) require much larger corpora of aligned data than we currently have, however some smarter algorithms such as chart-parsing (the CYK) algorithm with probabilistic grammars (PCFGs) should be usable already on the current scale of our data, perhaps complemented by leaner memory-based approaches such as k-nearest neighbor in the MBT toolkit.<sup>8</sup> Currently, we have started experimenting with the Stanford parser,<sup>9</sup> the Moses toolkit,<sup>10</sup> and our own Prolog/Perl implementation of the (lexicalized) CYK algorithm on a subset of 500 formal (bracketed) Flyspeck expressions about trigonometric functions. Such initial experiments concern relaxing of the precise disambiguated formal texts by adding more ambiguity. For example whenever a casting functor (such as  $Cx$  or  $\&$ ) has to be used in the formal text, we can remove it, and measure the success of the probabilistic parsing getting the right formal meaning. Once such experiments produce good results, the next step in this direction is learning the alignment of the informal/formal text/trees using for example the tree-based learning in the Moses toolkit. The work with established tools such as the Stanford parser and Moses will likely be complemented by our custom implementations that take advantage of the domain knowledge. For example we can add immediate pruning of potential parse trees in the CYK algorithm (or any chart parser) by using the HOL Light (Hindley-Milner) type system or the Mizar (soft, dependent) type system at each step of the algorithm.

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<sup>8</sup> <http://ilk.uvt.nl/mbt/>

<sup>9</sup> <http://nlp.stanford.edu/software/lex-parser.shtml>

<sup>10</sup> <http://www.statmt.org/moses/>